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Topic: special theory of relativity

Cochetkov Victor Nikolayevich
chief specialist FSUE "Center for exploitation of space ground-based infrastructure facilities" (FSUE "TSENKI")

vnkochetkov@gmail.com
vnkochetkov@rambler.ru
http://www.matphysics.ru

Using the law of conservation of momentum for test the validity of the special theory of relativity

The article attempts to show that the use of the special theory of relativity, when considering the motion of a closed mechanical system in the inertial reference systems, can lead to non-compliance with the law of conservation of momentum.

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Content

1. The introduction

Special theory of relativity can be divided into relativistic kinematics and relativistic dynamics.

The relativistic kinematics establishes a connection (Lorentz transformations) between the coordinates and time of an event, occurring at the point of the space, in one inertial reference system and coordinates and time of the same events in another inertial reference system, and the relationship between the values of projections of speeds of the point (conversion of the speeds) in appropriate times in two inertial reference systems.

The relativistic dynamics, based on the mandatory implementation of the laws of conservation of momentum and energy for the closed system of bodies, whose interaction is instantaneous in nature, in inertial reference systems, establishes the dependences of mass and momentum of the point material body from its speed.

The article suggests:

- to take a closed mechanical system of bodies, whose interaction will be permanent;
- to select two mobile and immobile inertial reference systems with respect to the center of mass of a closed system of bodies;
- to select two points in time in the mobile inertial reference system;
- with the help of the Lorentz transformation to determine the position of bodies in the selected points in time in the mobile reference system;
- using the conversion speeds to determine the projections of speeds of bodies in these moments of time in the mobile reference system;
- to determine the values of the momentums of the bodies in selected points in time in the mobile reference system, knowing the values of projections of speeds of the bodies and using the dependences of mass and momentum of a body on the speed;
- to write the law of conservation of momentum for the two selected points in time in the mobile reference system and to determine the conditions for its

implementation.

2. The main dependences of the special theory of relativity

Assume that there are two inertial reference systems, shown in Fig.1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$, in which:

- similar the axis of the Cartesian coordinate systems $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$ are pairs parallel and equally directed;

- system $O_2x_2y_2z_2$ moves relative to the system $O_1x_1y_1z_1$ with constant speed V along the axis O_1x_1;

- in both systems as the start timing (t_1=0 and t_2=0) is selected when the origin O_1 and O_2 of these systems are identical.

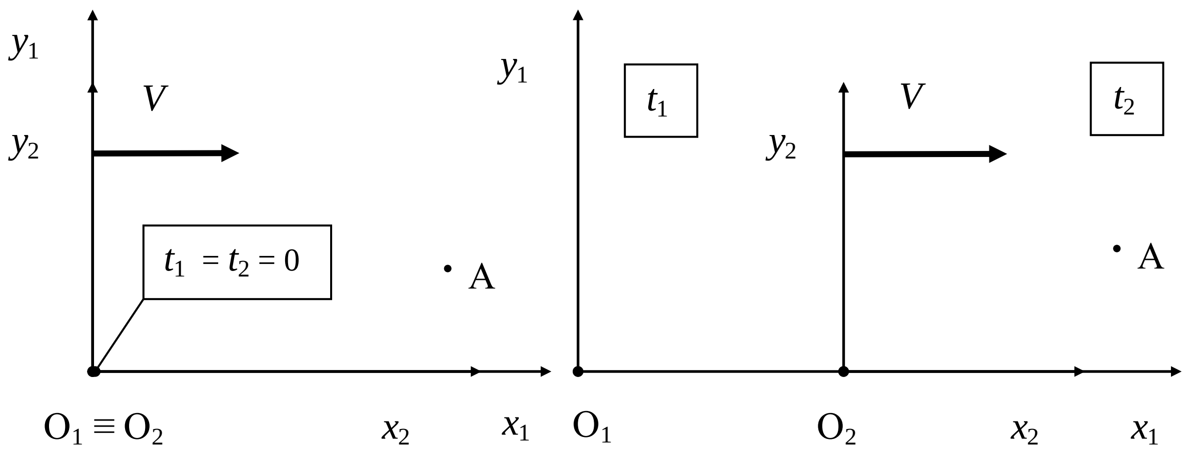

Fig.1

In the special theory of relativity, Lorentz transformations [1] - the relationship between the coordinates x_1, y_1, z_1 of point A at time t_1 in a stationary inertial reference system $O_1x_1y_1z_1$ and coordinates x_2, y_2, z_2 of the same point A in the mobile inertial reference system $O_2x_2y_2z_2$ at the time t_2, corresponding to time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$, as follows:

$$x_1 = \frac{x_2 + (V \cdot t_2)}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (1)$$

$$x_2 = \frac{x_1 - (V \cdot t_1)}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (2)$$

$$y_1 = y_2 \qquad (3)$$

$$z_1 = z_2 \qquad (4)$$

where: c - the speed of light in a vacuum.

From formulas (1) and (2) we can write the dependence for times t_1 and t_2 :

$$t_1 = \frac{t_2 + \frac{V \cdot x_2}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (5)$$

$$t_2 = \frac{t_1 - \frac{V \cdot x_1}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (6)$$

Also in the special theory of relativity, conversion of the speeds [1] - the relationship between the projections v_{x1}, v_{y1} and v_{z1} of the speed of a point on the axis of the Cartesian coordinates in time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$ and similar projections v_{x2}, v_{y2} and v_{z2} of the speed of the same point in the mobile inertial reference system $O_2x_2y_2z_2$ at time t_2, corresponding to time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$, written as:

$$v_{x1} = \frac{v_{x2} + V}{1 + \frac{V \cdot v_{x2}}{c^2}} \qquad (7)$$

$$v_{x2} = \frac{v_{x1} - V}{1 - \frac{V \cdot v_{x1}}{c^2}} \qquad (8)$$

$$v_{y1} = \frac{v_{y2} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V \cdot v_{x2}}{c^2}} \qquad (9)$$

$$v_{y2} = \frac{v_{y1} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V \cdot v_{x1}}{c^2}} \qquad (10)$$

…

The dependence [1] of the mass $M(v)$ and the momentum $\bar{P}(v)$ of a moving body, having a rest mass M_o , on the speed $\bar{v}$ in the special theory of relativity are taken in the forms:

$$M(v) \;=\; \frac{M_o}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (11)$$

$$\bar{P}(v) \;=\; \frac{M_o \cdot \bar{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (12)$$

3. The description of a closed mechanical system of bodies

For consideration we take the simplest closed mechanical system of the bodies, which have constant interaction.

Assume that there are two inertial reference systems, similar to those of reference systems, shown in Fig.1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$, which moves with speed V parallel to the axis O_1x_1 relative to the system $O_1x_1y_1z_1$.

Suppose that there is a closed mechanical system of bodies, shown in Fig.2 and consisting of point bodies 1 and 2, with equal mass M_o at rest, and a string 3.

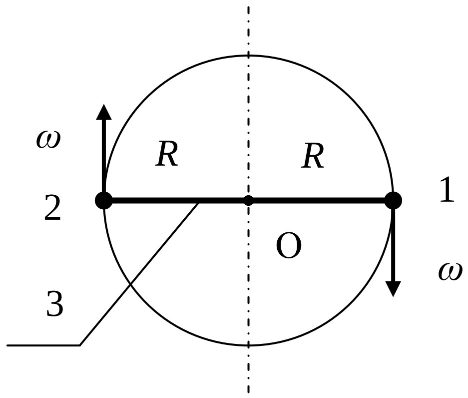

Fig.2

Bodies 1 and 2 are connected by a string 3, the mass of which can be neglected because of its smallness.

Bodies 1 and 2 rotate with angular speed ω around a common center of mass - the point O.

Distance from the point body 1 (body 2) to point O is equal to R.

Let's put a closed mechanical system of bodies 1 and 2 with a string 3 in the moving reference system $O_2x_2y_2z_2$ so, that the point O would be stationary in this reference system, and coincided with the origin O_2, and the rotation of bodies 1 and 2 around it would occur in a clockwise direction in the plane of $O_2x_2y_2$, as shown in Fig.3.

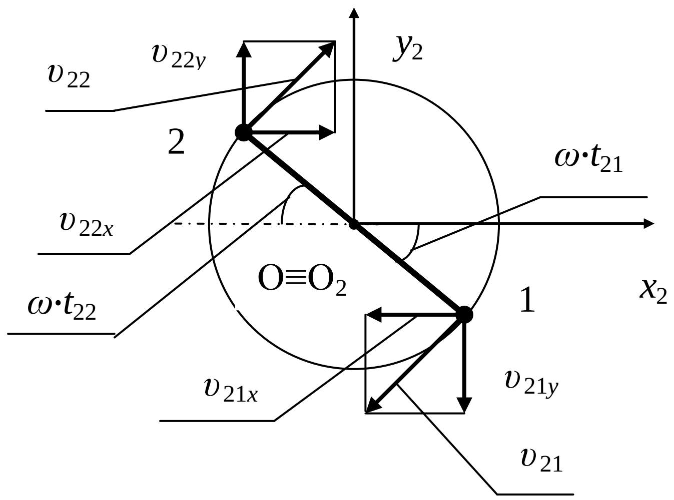

Fig.3

Also assume, that at the start of timing (t_2=0) in the reference system $O_2x_2y_2z_2$ bodies 1 and 2 were on the axis O_2x_2 , with the body 1 had a positive coordinate, and the body 2 - negative.

In the mobile reference system $O_2x_2y_2z_2$ at any time t_2 bodies 1 and 2 will have the speeds υ_{21} and υ_{22} , equal v_R :

$$v_{21} = v_{22} = v_R = \omega \cdot R \quad (13)$$

In this case, the projections υ_{21x} and υ_{21y} of speed of the body 1 and the projections υ_{22x} and υ_{22y} of speed of the body 2 on the axis O_2x_2 and O_2y_2 , respectively, for times t_{21} and t_{22} will be equal to:

$$v_{21\,x} = -\left[v_R \cdot \sin(\omega \cdot t_{21})\right] \quad (14)$$

$$v_{21\,y} = -\left[v_R \cdot \cos(\omega \cdot t_{21})\right] \quad (15)$$

$$v_{22x} = v_R \cdot \sin(\omega \cdot t_{22}) \quad (16)$$

$$v_{22y} = v_R \cdot \cos(\omega \cdot t_{22}) \quad (17)$$

The relationship between the coordinates x_{21} and y_{21} of the body 1 depending on time t_{21} and the relationship between the coordinates x_{22} and y_{22} of the body 2 depending on the time t_{22} in the mobile reference system $O_2x_2y_2z_2$ can be written as:

$$x_{21} = R \cdot \cos(\omega \cdot t_{21}) \quad (18)$$

$$y_{21} = -[R \cdot \sin(\omega \cdot t_{21})] \quad (19)$$

$$x_{22} = -[R \cdot \cos(\omega \cdot t_{22})] \quad (20)$$

$$y_{22} = R \cdot \sin(\omega \cdot t_{22}) \quad (21)$$

Based on the equations (1) and (3), we can write the relationships between:

- coordinates x_{11} and y_{11} of the body 1 at time t_{11} in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{21} and y_{21} of the body 1 in the mobile reference system $O_2x_2y_2z_2$ at time t_{21} , which corresponds to the time t_{11} in the stationary reference system $O_1x_1y_1z_1$:

$$x_{11} = \frac{x_{21} + (V \cdot t_{21})}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (22)$$

$$y_{11} = y_{21} \quad (23)$$

- coordinates x_{12} and y_{12} of the body 2 at time t_{12} in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{22} and y_{22} of the body 2 in the mobile reference system $O_2x_2y_2z_2$ at time t_{22} , which corresponds to the time t_{12} in the stationary reference system $O_1x_1y_1z_1$:

$$x_{12} = \frac{x_{22} + (V \cdot t_{22})}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (24)$$

$$y_{12} = y_{22} \quad (25)$$

Using formula (5) relationship between the values of the times t_{11} and t_{21} , t_{12} and t_{22} will look like this:

$$t_{11} = \frac{t_{21} + \frac{V \cdot x_{21}}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (26)$$

$$t_{12} = \frac{t_{22} + \frac{V \cdot x_{22}}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (27)$$

In considering we are interested in the position of bodies 1 and 2 in the stationary reference system $O_1x_1y_1z_1$ at the same time, ie where:

$$t_{11} = t_{12} \qquad (28)$$

The equation (28) taking into account formulas (18), (20), (26) and (27) becomes:

$$t_{21} + \frac{V \cdot R \cdot \cos(\omega \cdot t_{21})}{c^2} = t_{22} - \frac{V \cdot R \cdot \cos(\omega \cdot t_{22})}{c^2} \qquad (29)$$

Now for consideration, select two points in time in the stationary reference system $O_1x_1y_1z_1$.

4. Moment of time t_{1p}

In the mobile reference system $O_2x_2y_2z_2$ when performing the condition (28) it is interesting position of the bodies 1 and 2 at the time t_{2p} , when:

$$t_{21} = t_{22} = t_{2p} \qquad (30)$$

Substituting condition (30) in equation (29) for the case when $(\omega \cdot t_{2p}) < \pi$, we obtain:

$$\omega \cdot t_{2p} = \frac{\pi}{2} \qquad (31)$$

Ie, as shown in Fig.4, under the terms of (28), (30) and (31) in the moving mobile reference system $O_2x_2y_2z_2$ at time t_{2p} the bodies 1 and 2 are on a line parallel to the axis O_2y_2 and in the stationary reference system $O_1x_1y_1z_1$ the bodies 1 and 2 will be on a line parallel to the axis O_1y_1 at time t_{11} (t_{12}), equal t_{1p} and which corresponds to the time t_{2p} in the mobile reference system $O_2x_2y_2z_2$.

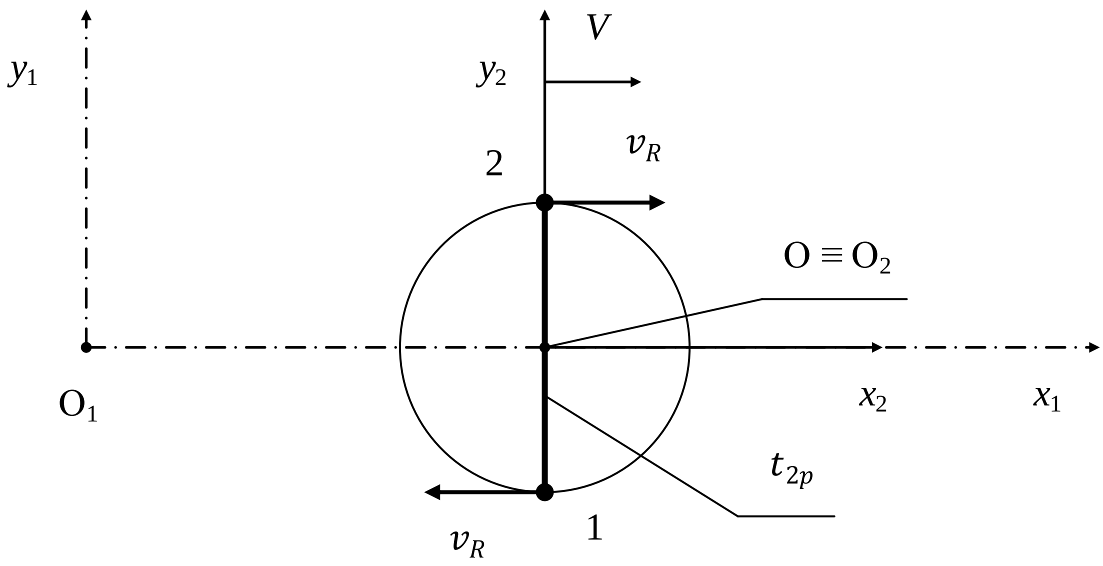

Fig.4

According to equations (31), (14) - (17) in the mobile reference system $O_2x_2y_2z_2$ at time t_{2p} the bodies 1 and 2, respectively, have the following values of the projections $v_{21\,xp}$, $v_{21\,yp}$ and $v_{22\,xp}$, $v_{22\,yp}$ of speeds of his movement on the axis O_2x_2 and O_2y_2 :

$$v_{21\,xp} = -\,v_R \qquad (32)$$

$$v_{21\,yp} = 0 \qquad (33)$$

$$v_{22\,xp} = v_R \qquad (34)$$

$$v_{22\,yp} = 0 \qquad (35)$$

Then, on the basis of formulas (7), (9) and equalities (32) - (35), in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} the body 1 and the body 2, respectively, will have the following values of the projections v_{11xp} , $v_{11\,yp}$ and $v_{12\,xp}$, $v_{12\,yp}$ of speeds of his movement on the axis O_1x_1 and O_1y_1 :

$$v_{11\,xp} = \frac{V - v_R}{1 - \dfrac{V \cdot v_R}{c^2}} \qquad (36)$$

$$v_{11\,yp} = 0 \qquad (37)$$

$$v_{12\,xp} = \frac{V + v_R}{1 + \dfrac{V \cdot v_R}{c^2}} \qquad (38)$$

$$v_{12\,yp} = 0 \qquad (39)$$

Hence, using formulas (11) and (12), may be noted that in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} the body 1 and the body 2, respectively, will have the following values of the projections $P_{11\,xp}$, $P_{11\,yp}$ and $P_{12\,xp}$, $P_{12\,yp}$ of momentums on the axis O_1x_1 and O_1y_1:

$$P_{11\,xp} \;=\; \frac{M_{\text{o}} \cdot v_{11\,x\text{p}}}{\sqrt{1-\frac{v_{11\,x}^2}{c^2}}} \qquad (40)$$

$$P_{12\,xp} \;=\; \frac{M_{\text{o}} \cdot v_{12\,x\text{p}}}{\sqrt{1-\frac{v_{12\,x}^2}{c^2}}} \qquad (41)$$

$$P_{11\,yp} \;=\; 0 \qquad (42)$$

$$P_{12\,yp} \;=\; 0 \qquad (43)$$

5. Moment of time t_{1h}

Also in the mobile reference system $O_2x_2y_2z_2$ when performing the condition (28) it is interesting position of body 2 when finding the body 1 on the axis O_2x_2 at time t_{21} , equal to $t_{21\,h}$, where:

$$t_{21\,h} = 0 \qquad (44)$$

The value of time t_{22} , when performing the conditions (28) and (44), denote t_{22h} , for which the equation (29) becomes:

$$\omega \cdot t_{22\,h} = \frac{v_R \cdot V}{c^2} \cdot \;[1 + \;\cos(\omega \cdot t_{22\,h})] \;\cdot \qquad (45)$$

As can be seen from equation (45), the value of time t_{22h} must be greater than 0.

Under the terms of (28) and (44) in the mobile reference system $O_2x_2y_2z_2$ at time $t_{21\,h} = 0$ the body 1 will be located on the axis O_2x_2 , and in the stationary reference system $O_1x_1y_1z_1$ the body 1 will be located on the axis O_1x_1 at time t_{11} (t_{12}), equal t_{1h} and which corresponds to the time $t_{21\,h} = 0$ in the mobile reference system $O_2x_2y_2z_2$.

Moreover in the mobile reference system $O_2x_2y_2z_2$ according to equation (45), the body 2 can not be on the axis O_2x_2 at time t_{22} , equal $t_{22\,h}$ and which corresponds to the time t_{1h} in the stationary reference system $O_1x_1y_1z_1$.

Ie, as shown in Fig.5, the body 1 is located on the axis O_1x_1 in a the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} , which corresponds to the time $t_{21h} = 0$ in the mobile reference system $O_2x_2y_2z_2$, and at the time t_{1h} the body 2 can not lie on the axis O_2x_2 .

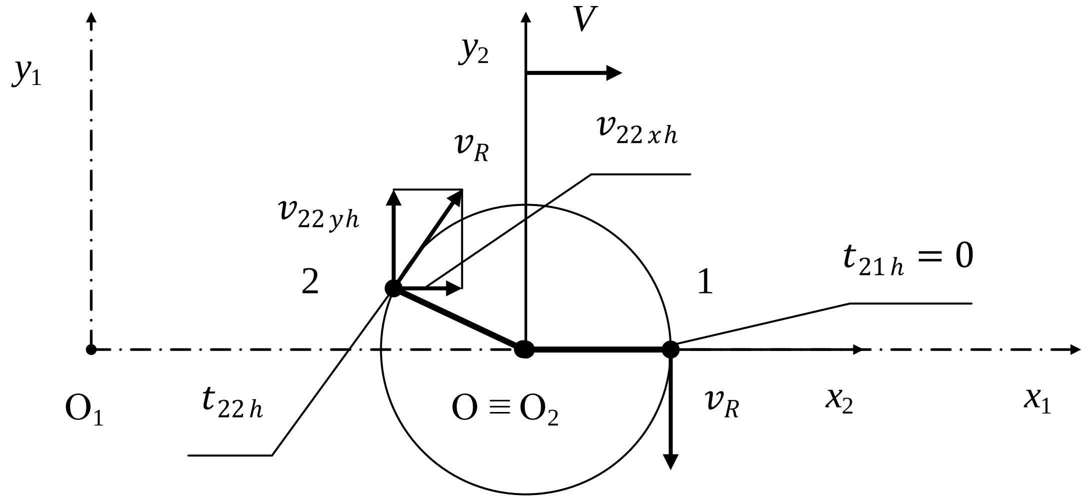

Fig.5

In the mobile reference system $O_2x_2y_2z_2$ the body 1 at the time $t_{21h} = 0$ and the body 2 at the time t_{22h} respectively have projections v_{21xh} , v_{21yh} and v_{22xh} , v_{22yh} of speeds of his movement on the axis O_2x_2 and O_2y_2 , so that:

$$v_{21xh} = 0 \qquad (46)$$

$$v_{21yh} = -v_R \qquad (47)$$

Then, on the basis of formulas (7), (9) and equations (46), (47), in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} the body 1 and the body 2, respectively, will have the values of the projections v_{11xh} , v_{11yh} and v_{12xh} , v_{12yh} of speeds of his movement on the axis O_1x_1 and O_1y_1 :

$$v_{11xh} = V \qquad (48)$$

$$v_{11yh} = -\left(v_R \cdot \sqrt{1 - \frac{V^2}{c^2}}\right) \qquad (49)$$

$$v_{12xh} = \frac{V + v_{22xh}}{1 + \frac{V \cdot v_{22xh}}{c^2}} \qquad (50)$$

$$v_{12yh} = \frac{v_{22yh} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V \cdot v_{22xh}}{c^2}} \qquad (51)$$

Given equation (45), we note that, ie the time $t_{22h} > 0$, so the projection v_{22yh} of the speed will be the direction of the axis O_2y_2.

From equations (16) and (17) it follows that:

$$v_{22xh}{}^2 + v_{22yh}{}^2 = v_R{}^2 \qquad (52)$$

Using formulas (11) and (12), may be noted that in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} the body 1 and the body 2, respectively, will have the following values of the projections P_{11xh} , P_{11yh} and P_{12xh} , P_{12yh} of momentums on the axis O_1x_1 and O_1y_1:

$$P_{11xh} = \frac{M_o \cdot v_{11xh}}{\sqrt{1 - \frac{v_{11xh}^2 + v_{11yh}^2}{c^2}}} \qquad (53)$$

$$P_{12xh} = \frac{M_o \cdot v_{12xh}}{\sqrt{1 - \frac{v_{12xh}^2 + v_{12yh}^2}{c^2}}} \qquad (54)$$

$$P_{11yh} = \frac{M_o \cdot v_{11yh}}{\sqrt{1 - \frac{v_{11xh}^2 + v_{11yh}^2}{c^2}}} \qquad (55)$$

$$P_{12yh} = \frac{M_o \cdot v_{12yh}}{\sqrt{1 - \frac{v_{12xh}^2 + v_{12yh}^2}{c^2}}} \qquad (56)$$

6. The verification of the law of conservation of momentum

The law of conservation of momentum of a closed mechanical system of bodies, connected with the symmetry properties of space - the homogeneity of space [1], states, that the momentum of a closed mechanical system of bodies (which is not acted upon by external forces) is a constant value, ie in any inertial reference system for any point in time the value of the momentum of a closed mechanical system of bodies is a constant value (because there is no external influence).

Due to the fact, that the mechanical system of the bodies 1 and 2 (and string 3) is closed, the law of conservation of momentum can write the following equations for the moments of times t_{1p} and t_{1h} :

$$P_{11xp} + P_{12xp} = P_{11xh} + P_{12xh}$$
$$P_{11yp} + P_{12yp} = P_{11yh} + P_{12yh}$$

or:

$$\frac{M_o \cdot v_{11xp}}{\sqrt{1-\frac{v_{11xp}^2}{c^2}}} + \frac{M_o \cdot v_{12xp}}{\sqrt{1-\frac{v_{12xp}^2}{c^2}}} = \frac{M_o \cdot v_{11xh}}{\sqrt{1-\frac{v_{11xh}^2+v_{11yh}^2}{c^2}}} + \frac{M_o \cdot v_{12xh}}{\sqrt{1-\frac{v_{12xh}^2+v_{12yh}^2}{c^2}}} \quad (57)$$

$$0 = \frac{M_o \cdot v_{11yh}}{\sqrt{1-\frac{v_{11xh}^2+v_{11yh}^2}{c^2}}} + \frac{M_o \cdot v_{12yh}}{\sqrt{1-\frac{v_{12xh}^2+v_{12yh}^2}{c^2}}} \quad (58)$$

By inserting the projections v_{11xp} , v_{12xp} , v_{11xh} , v_{11yh} , v_{12xh} and v_{12yh} of speeds of formulas (36), (38), (48) - (51) in equations (57) and (58) and using the formula (52), we obtain:

$$\frac{M_o \cdot (V - v_R)}{\sqrt{1-\frac{{v_R}^2}{c^2}} \cdot \sqrt{1-\frac{V^2}{c^2}}} + \frac{M_o \cdot (V + v_R)}{\sqrt{1-\frac{{v_R}^2}{c^2}} \cdot \sqrt{1-\frac{V^2}{c^2}}} =$$

$$= \frac{M_o \cdot V}{\sqrt{1-\frac{{v_R}^2}{c^2}} \cdot \sqrt{1-\frac{V^2}{c^2}}} + \frac{M_o \cdot (V + v_{22xh})}{\sqrt{1-\frac{{v_R}^2}{c^2}} \cdot \sqrt{1-\frac{V^2}{c^2}}} \quad (59)$$

$$0 = -\frac{M_o \cdot v_R}{\sqrt{1-\frac{{v_R}^2}{c^2}}} + \frac{M_o \cdot v_{22yh}}{\sqrt{1-\frac{{v_R}^2}{c^2}}} \quad (60)$$

or:

$$V - v_R + V + v_R = V + V + v_{22xh} \quad (61)$$

$$0 = -v_R + v_{22yh} \tag{62}$$

From equations (61) and (62) obtain the necessary conditions (the values of the projections v_{22xh} and v_{22yh} of speeds), which in this example will be implemented by law of conservation of momentum in the stationary inertial reference system $O_1x_1y_1z_1$:

$$v_{22xh} = 0 \tag{63}$$

$$v_{22yh} = v_R \tag{64}$$

Substituting conditions (63) and (64) in equations (16) and (17), we obtain:

$$t_{22h} = t_{21h} = 0 \tag{65}$$

And substituting equation (65) in the formula (45):

$$\omega \cdot 0 = \frac{v_R \cdot V}{c^2} \cdot [1 + 1] \tag{66}$$

will have another condition for the implementation of the law of conservation of momentum in the stationary inertial reference system $O_1x_1y_1z_1$ for considered example:

$$0 = \frac{1}{c^2} \tag{67}$$

But since the speed of light c is not infinite, so the condition (67) is not feasible, and therefore in this case, the law of conservation of momentum can not be implemented.

Ie, we can conclude, that in the stationary inertial reference system $O_1x_1y_1z_1$ the application of the special theory of relativity to describe the motion of a closed mechanical system of bodies, considered in this example, leads to non-compliance of the law of conservation of momentum.

9. Conclusion

It can be concluded, that the use of the special theory of relativity in dealing with individual examples may lead to non-compliance with the law of conservation of momentum for a closed mechanical system in the inertial reference systems.

Given, that the law of conservation of momentum associated with the homogeneity of space [1], we can assume, that the failure of the law of conservation of momentum will lead to non-compliance with conditions of symmetry of space and time, on which is based the special theory of relativity.

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Author V.N. Cochetkov

Author - Cochetkov Victor Nikolayevich

E-mail: VNKochetkov@gmail.com .

E-mail: VNKochetkov@rambler.ru .

Website: http://www.matphysics.ru .