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Special Relativity: Depending on the Definition of the Momentum of a Closed System of Bodies from Time

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ABSTRACT: The article attempts to show a concrete example, that the application of the special theory of relativity, when considering the motion of a closed mechanical system of bodies in inertial reference systems, can lead to the fact, that the momentum of a closed system will be a function of time.

KEYWORDS: The special theory of a relativity, the law of conservation of momentum of the closed mechanical system, dependence of weight of a body on speed of its movement, symmetry of space and time, relativity principle.

I. INTRODUCTION

In special relativity the dependence of the point body of mass from its speed is determined from the condition of the mandatory implementation of the laws of conservation of momentum and energy for the closed system of bodies, whose interaction is instantaneous in nature.

In this article I propose to use as an example a closed mechanical system of bodies, whose interaction is ongoing, to confirm the applicability of the law of conservation of momentum in the case using the special theory of relativity.

II. DESCRIPTION OF A CLOSED MECHANICAL SYSTEM OF BODIES

For consideration we take the simplest closed mechanical system of bodies that undergo constant interaction.

Suppose that there is a closed mechanical system of bodies, shown in Fig.1 and consisting of point bodies 1 and 2, with equal mass M_0 at rest, and a string 3.

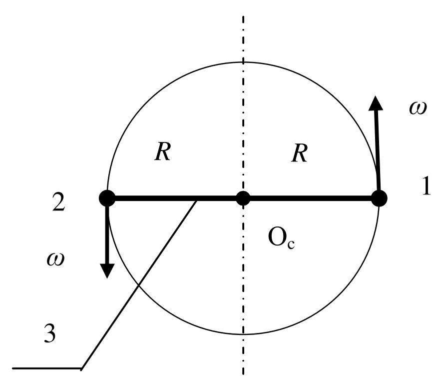

Fig.1

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Bodies 1 and 2 are connected by a string 3, which has a mass of uniformly distributed along its length and equal to m_o at rest.

Bodies 1 and 2 rotate with angular speed ω around a common center of mass - the point O_c.

Distance from the point body 1 (body 2) to point O_c is equal to R.

Let's put a closed mechanical system of bodies 1 and 2 with a string 3 in the moving reference system O*xyz* so, that the point O_c would be stationary in this reference system, and coincided with the origin O, and the rotation of bodies 1 and 2 around it would occur in a counter-clockwise direction in the plane of O*xy*, as shown in Fig.2.

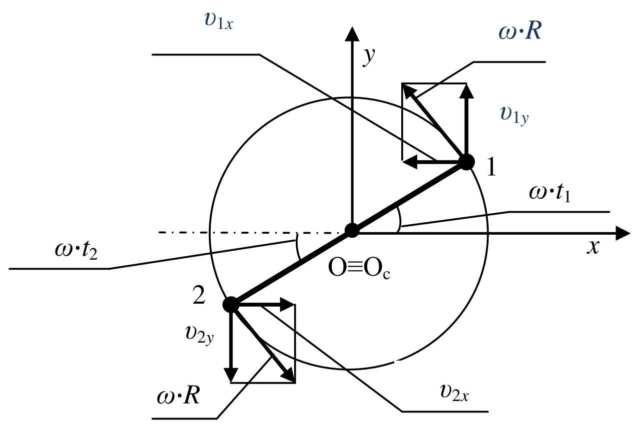

Fig.2

Also assume, that at the start of timing (t=0) in the reference system O*xyz* bodies 1 and 2 were on the axis O*x*, with the body 1 had a positive coordinate, and the body 2 - negative.

In the reference system O*xyz* :

- body 1 has the coordinates x_1 and y_1 and projection v_{1x} and v_{1y} of the speed on the axis O*x* and O*y*, respectively, depending on the time t_1:

$$x_1 = R \cdot \cos(\omega \cdot t_1) \quad (1)$$

$$y_1 = R \cdot \sin(\omega \cdot t_1) \quad (2)$$

$$v_{1x} = -[\omega \cdot R \cdot \sin(\omega \cdot t_1)] \quad (3)$$

$$v_{1y} = [\omega \cdot R \cdot \cos(\omega \cdot t_1)] \quad (4)$$

- body 2 has the coordinates x_2 and y_2 and projection v_{2x} and v_{2y} of the speed on the axis O*x* and O*y*, respectively, depending on the time t_2:

$$x_2 = -[R \cdot \cos(\omega \cdot t_2)] \quad (5)$$

$$y_2 = -[R \cdot \sin(\omega \cdot t_2)] \quad (6)$$

$$v_{2x} = \omega \cdot R \cdot \sin(\omega \cdot t_2) \quad (7)$$

$$v_{2y} = -[\omega \cdot R \cdot \cos(\omega \cdot t_2)] \quad (8)$$

Let's introduce another inertial reference system O'*x*'*y*'*z*', shown in Fig.3.

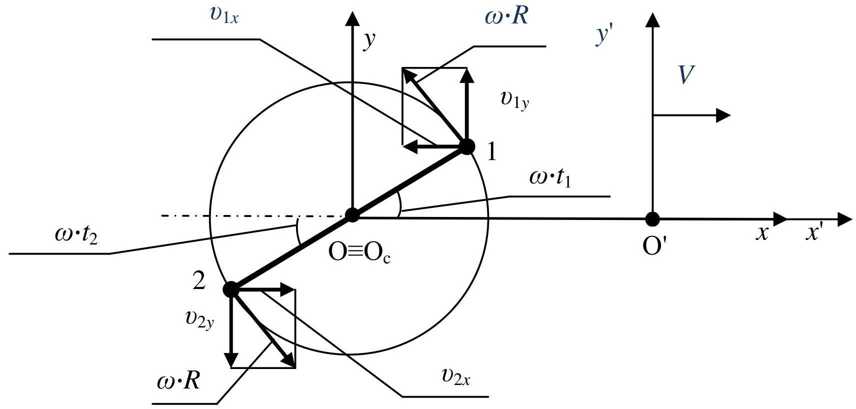

Fig.3

Assume that the inertial reference systems O*xyz* and O'*x*'*y*'*z*':

- similar the axis of the Cartesian coordinate are pairs parallel and equally directed;
- system O'*x*'*y*'*z*' moves relative to the system O*xyz* with constant speed *V* along the axis O*x*;
- in both systems as the start timing (t=0 and t'=0) is selected, when the origin O and O' of these systems are identical.

Based on the Lorentz transformation and conversion of the speeds [1] we can write:

- the relationship between coordinates x'_1 and y'_1 of the body 1 at time t'_1 in the reference system O'*x*'*y*'*z*' and the coordinates x_1 and y_1 of the body 1 in the reference system O*xyz* at time t_1, the corresponding time t'_1 in the reference system O'*x*'*y*'*z*':

$$x'_1 = \frac{x_1 - (V \cdot t_1)}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (9)$$

$$y'_1 = y_1 \qquad (10)$$

- the relationship between the moment of time t'_1 in the reference system O'*x*'*y*'*z*' and the time t_1 in the reference system O*xyz*, the corresponding time t'_1 in the reference system O'*x*'*y*'*z*':

$$t'_1 = \frac{t_1 - \frac{V \cdot x_1}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{t_1 - \frac{V \cdot R \cdot \cos(\omega \cdot t_1)}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (11)$$

- the relationship between the projections v'_{x1} and v'_{y1} of the speed v'_1 of the body 1 on the axis of the Cartesian coordinates at time t'_1 in the reference system O'*x*'*y*'*z*' and projections v_{x1} and v_{y1} of the speed v_1 of the body 1on the axis of the Cartesian coordinate in the reference system O*xyz* at time t_1, the corresponding time t'_1 in the reference system O'*x*'*y*'*z*':

$$v'_{x1} = \frac{v_{x1} - V}{1 - \frac{V \cdot v_{x1}}{c^2}} = -\frac{[\omega \cdot R \cdot \sin(\omega \cdot t_1)] + V}{1 + \frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_1)}{c^2}} \quad (12)$$

$$v'_{y1} = \frac{v_{y1} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V \cdot v_{x1}}{c^2}} = \frac{\omega \cdot R \cdot \cos(\omega \cdot t_1) \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_1)}{c^2}} \quad (13)$$

moreover:

$$v'^2_1 = v'^2_{x1} + v'^2_{y1} = \frac{\left\{1 + \frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_1)}{c^2}\right\}^2 - \left[\left(1 - \frac{V^2}{c^2}\right) \cdot \left(1 - \frac{\omega^2 \cdot R^2}{c^2}\right)\right]}{\left\{1 + \frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_1)}{c^2}\right\}^2} \cdot c^2 \quad (14)$$

- the relationship between coordinates x'_2 and y'_2 of the body 2 at time t'_2 in the reference system O'*x*'*y*'*z*' and the coordinates x_2 and y_2 of the body 2 in the reference system O*xyz* at time t_2, the corresponding time t'_2 in the reference system O'*x*'*y*'*z*':

$$x'_2 = \frac{x_2 - (V \cdot t_2)}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (15)$$

$$y'_2 = y_2 \quad (16)$$

- the relationship between the moment of time t'_2 in the reference system O'*x*'*y*'*z*' and the time t_2 in the reference system O*xyz*, the corresponding time t'_2 in the reference system O'*x*'*y*'*z*':

$$t'_2 = \frac{t_2 - \frac{V \cdot x_2}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{t_2 + \frac{V \cdot R \cdot \cos(\omega \cdot t_2)}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (17)$$

- the relationship between the projections v'_{x2} and v'_{y2} of the speed v'_2 of the body 2 on the axis of the Cartesian coordinates at time t'_2 in the reference system O'*x*'*y*'*z*' and projections v_{x2} and v_{y2} of the speed v_2 of the body 2 on the axis of the Cartesian coordinate in the reference system O*xyz* at time t_2, the corresponding time t'_2 in the reference system O'*x*'*y*'*z*':

$$v'_{x2} = \frac{v_{x2} - V}{1 - \frac{V \cdot v_{x2}}{c^2}} = \frac{[\omega \cdot R \cdot \sin(\omega \cdot t_2)] - V}{1 - \frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_2)}{c^2}} \quad (18)$$

$$v'_{y2} = \frac{v_{y2} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V \cdot v_{x2}}{c^2}} = -\frac{\omega \cdot R \cdot \cos(\omega \cdot t_2) \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_2)}{c^2}} \quad (19)$$

moreover:

$$v'^2_2 = v'^2_{x2} + v'^2_{y2} = \frac{\left\{1 - \frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_2)}{c^2}\right\}^2 - \left[\left(1 - \frac{V^2}{c^2}\right) \cdot \left(1 - \frac{\omega^2 \cdot R^2}{c^2}\right)\right]}{\left\{1 - \frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_2)}{c^2}\right\}^2} \cdot c^2 \quad (20)$$

To address the string 3 in a state of rest be divided into 2·n equal parts, with accommodation in the center of each part the point body with rest mass m_{0n}, equal to:

$$m_{0n} = \frac{m_0}{2 \cdot n} \qquad (21)$$

Points of the string 3, located on the segment from the point O_c to the body 1, denoted as *i*-points (*i* = 0, 1, 2, 3, ... n), and points of the string 3, located on the segment from the point O_c to body 2, denoted as the *j*-points (*j* = 0, 1, 2, 3, ... n).

The distance R_i from the point O_c to *i*-the point of string 3 will take:

$$R_i = R \cdot \left(\frac{1}{2 \cdot n} + \frac{i-1}{n}\right) \qquad (22)$$

The distance R_j from the point O_c to *j*-the point of string 3 will take:

$$R_j = R \cdot \left(\frac{1}{2 \cdot n} + \frac{j-1}{n}\right) \qquad (23)$$

In the reference system O*xyz* :

- the *i*-point of string 3 has the coordinates x_{1i} and y_{1i} and projection υ_{1xi} and υ_{1yi} of the speed on the axis O*x* and O*y*, respectively, depending on the time t_{1i}:

$$x_{1i} = R_i \cdot \cos(\omega \cdot t_{1i}) \qquad (24)$$

$$y_{1i} = R_i \cdot \sin(\omega \cdot t_{1i}) \qquad (25)$$

$$v_{1xi} = -[\omega \cdot R_i \cdot \sin(\omega \cdot t_{1i})] \qquad (26)$$

$$v_{1yi} = [\omega \cdot R_i \cdot \cos(\omega \cdot t_{1i})] \qquad (27)$$

- the *j*-point of string 3 has the coordinates x_{2j} and y_{2j} and projection υ_{2xj} and υ_{1yj} of the speed on the axis O*x* and O*y*, respectively, depending on the time t_{2j}:

$$x_{2j} = -R_j \cdot \cos(\omega \cdot t_{2j}) \qquad (28)$$

$$y_{2j} = -R_j \cdot \sin(\omega \cdot t_{2j}) \qquad (29)$$

$$v_{2xj} = \omega \cdot R_j \cdot \sin(\omega \cdot t_{2j}) \qquad (30)$$

$$v_{2yj} = -[\omega \cdot R_j \cdot \cos(\omega \cdot t_{2j})] \qquad (31)$$

Similarly, using the Lorentz transformation and conversion of the speeds [1] we can write:

- the relationship between coordinates x'_{1i} and y'_{1i} of the *i*-the point of string 3 at time t'_{1i} in the reference system O'*x*'*y*'*z*' and the coordinates x_{1i} and y_{1i} of the *i*-the point of string 3 in the reference system O*xyz* at time t_{1i}, the corresponding time t'_{1i} in the reference system O'*x*'*y*'*z*':

$$x'_{1i} = \frac{x_{1i} - (V \cdot t_{1i})}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (32)$$

$$y'_{1i} = y_{1i} \qquad (33)$$

- the relationship between the moment of time t'_{1i} in the reference system O'*x*'*y*'*z*' and the time t_{1i} in the reference system O*xyz*, the corresponding time t'_{1i} in the reference system O'*x*'*y*'*z*':

$$t'_{1i} = \frac{t_{1i} - \dfrac{V \cdot x_{1i}}{c^2}}{\sqrt{1 - \dfrac{V^2}{c^2}}} = \frac{t_{1i} - \dfrac{V \cdot R_i \cdot \cos(\omega \cdot t_{1i})}{c^2}}{\sqrt{1 - \dfrac{V^2}{c^2}}} \quad (34)$$

- the relationship between the projections v'_{x1i} and v'_{y1i} of the speed v'_{1i} of the *i*-the point of string 3 on the axis of the Cartesian coordinates at time t'_{1i} in the reference system O'*x'y'z'* and projections v_{x1i} and v_{y1i} of the speed v_{1i} of the *i*-the point of string 3 on the axis of the Cartesian coordinate in the reference system O*xyz* at time t_{1i}, the corresponding time t'_{1i} in the reference system O'*x'y'z'*:

$$v'_{x1i} = \frac{v_{x1i} - V}{1 - \dfrac{V \cdot v_{x1i}}{c^2}} = -\frac{[\omega \cdot R_i \cdot \sin(\omega \cdot t_{1i})] + V}{1 + \dfrac{V \cdot \omega \cdot R_i \cdot \sin(\omega \cdot t_{1i})}{c^2}} \quad (35)$$

$$v'_{y1i} = \frac{v_{y1i} \cdot \sqrt{1 - \dfrac{V^2}{c^2}}}{1 - \dfrac{V \cdot v_{x1i}}{c^2}} = \frac{\omega \cdot R_i \cdot \cos(\omega \cdot t_{1i}) \cdot \sqrt{1 - \dfrac{V^2}{c^2}}}{1 + \dfrac{V \cdot \omega \cdot R_i \cdot \sin(\omega \cdot t_{1i})}{c^2}} \quad (36)$$

moreover:

$$v'^2_{1i} = v'^2_{x1i} + v'^2_{y1i} = \frac{\left\{1 + \dfrac{V \cdot \omega \cdot R_i \cdot \sin(\omega \cdot t_{1i})}{c^2}\right\}^2 - \left[\left(1 - \dfrac{V^2}{c^2}\right) \cdot \left(1 - \dfrac{\omega^2 \cdot R_i^2}{c^2}\right)\right]}{\left\{1 + \dfrac{V \cdot \omega \cdot R_i \cdot \sin(\omega \cdot t_{1i})}{c^2}\right\}^2} \cdot c^2 \quad (37)$$

- the relationship between coordinates x'_{2j} and y'_{2j} of the *j*-the point of string 3 at time t'_{2j} in the reference system O'*x'y'z'* and the coordinates x_{2j} and y_{2j} of the *j*-the point of string 3 in the reference system O*xyz* at time t_{2j}, the corresponding time t'_{2j} in the reference system O'*x'y'z'*:

$$x'_{2j} = \frac{x_{2j} - (V \cdot t_{2j})}{\sqrt{1 - \dfrac{V^2}{c^2}}} \quad (38)$$

$$y'_{2j} = y_{2j} \quad (39)$$

- the relationship between the moment of time t'_{2j} in the reference system O'*x'y'z'* and the time t'_{2j} in the reference system O*xyz*, the corresponding time t'_{2j} in the reference system O'*x'y'z'*:

$$t'_{2j} = \frac{t_{2j} - \dfrac{V \cdot x_{2j}}{c^2}}{\sqrt{1 - \dfrac{V^2}{c^2}}} = \frac{t_{2j} + \dfrac{V \cdot R_j \cdot \cos(\omega \cdot t_{2j})}{c^2}}{\sqrt{1 - \dfrac{V^2}{c^2}}} \quad (40)$$

- the relationship between the projections v'_{x2j} and v'_{y2j} of the speed v'_{2j} of the *j*-the point of string 3 on the axis of the Cartesian coordinates at time t'_{2j} in the reference system O'*x'y'z'* and projections v_{x2j} and v_{y2j} of the speed v_{2j} of the *j*-the point of string 3 on the axis of the Cartesian coordinate in the reference system O*xyz* at time t_{2j}, the corresponding time t'_{2j} in the reference system O'*x'y'z'*:

$$v'_{x2j} = \frac{v_{x2j} - V}{1 - \dfrac{V \cdot v_{x2j}}{c^2}} = \frac{[\omega \cdot R_j \cdot \sin(\omega \cdot t_{2j})] - V}{1 - \dfrac{V \cdot \omega \cdot R_j \cdot \sin(\omega \cdot t_{2j})}{c^2}} \quad (41)$$

$$v'_{y2j} = \frac{v_{y2j} \cdot \sqrt{1-\frac{V^2}{c^2}}}{1-\frac{V \cdot v_{x2j}}{c^2}} = -\frac{\omega \cdot R_j \cdot \cos(\omega \cdot t_{2j}) \cdot \sqrt{1-\frac{V^2}{c^2}}}{1-\frac{V \cdot \omega \cdot R_j \cdot \sin(\omega \cdot t_{2j})}{c^2}} \quad (42)$$

moreover:

$$v'^{\,2}_{2j} = v'^{\,2}_{x2j} + v'^{\,2}_{y2j} = \frac{\left\{1-\frac{V \cdot \omega \cdot R_j \cdot \sin(\omega \cdot t_{2j})}{c^2}\right\}^2 - \left[\left(1-\frac{V^2}{c^2}\right)\cdot\left(1-\frac{\omega^2 \cdot R_j^2}{c^2}\right)\right]}{\left\{1-\frac{V \cdot \omega \cdot R_j \cdot \sin(\omega \cdot t_{2j})}{c^2}\right\}^2} \cdot c^2 \quad (43)$$

III. GETTING THE MOMENTUM EQUATION SYSTEM

Using the dependence of the mass of the moving body from its speed [1], we can write the following formula:

- formulas for the projections P'_{x1} and P'_{y1} of the momentum P'_1 of the body 1on the axis of the Cartesian coordinates in the reference system O'*x*'*y*'*z*' at time t'_1, the corresponding time t_1 in the reference system O*xyz*:

$$P'_{x1} = \frac{v'_{x1} \cdot M_0}{\sqrt{1-\frac{v'^{\,2}_1}{c^2}}} = -\frac{M_0 \cdot \{[\omega \cdot R \cdot \sin(\omega \cdot t_1)] + V\}}{\sqrt{1-\frac{V^2}{c^2}} \cdot \sqrt{1-\frac{\omega^2 \cdot R^2}{c^2}}} \quad (44)$$

$$P'_{y1} == \frac{v'_{y1} \cdot M_0}{\sqrt{1-\frac{v'^{\,2}_1}{c^2}}} = \frac{M_0 \cdot \omega \cdot R \cdot \cos(\omega \cdot t_1)}{\sqrt{1-\frac{\omega^2 \cdot R^2}{c^2}}} \quad (45)$$

$$P'^{\,2}_1 = P'^{\,2}_{x1} + P'^{\,2}_{y1} = M_0^2 \cdot c^2 \cdot \left[\frac{\left\{1+\frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_1)}{c^2}\right\}^2}{\left(1-\frac{V^2}{c^2}\right)\cdot\left(1-\frac{\omega^2 \cdot R^2}{c^2}\right)} - 1\right] \quad (46)$$

- formulas for the projections P'_{x2} and P'_{y2} of the momentum P'_2 of the body 2 on the axis of the Cartesian coordinates in the reference system O'*x*'*y*'*z*' at time t'_2, the corresponding time t_2 in the reference system O*xyz*:

$$P'_{x2} = \frac{v'_{x2} \cdot M_0}{\sqrt{1-\frac{v'^{\,2}_2}{c^2}}} = \frac{M_0 \cdot \{[\omega \cdot R \cdot \sin(\omega \cdot t_2)] - V\}}{\sqrt{1-\frac{V^2}{c^2}} \cdot \sqrt{1-\frac{\omega^2 \cdot R^2}{c^2}}} \quad (47)$$

$$P'_{y2} = \frac{v'_{y2} \cdot M_0}{\sqrt{1-\frac{v'^{\,2}_2}{c^2}}} = -\frac{M_0 \cdot \omega \cdot R \cdot \cos(\omega \cdot t_2)}{\sqrt{1-\frac{\omega^2 \cdot R^2}{c^2}}} \quad (48)$$

$$P'^{\,2}_2 = P'^{\,2}_{x2} + P'^{\,2}_{y2} = M_0^2 \cdot c^2 \cdot \left[\frac{\left\{1-\frac{V \cdot \omega \cdot R \cdot \sin(\omega \cdot t_2)}{c^2}\right\}^2}{\left(1-\frac{V^2}{c^2}\right)\cdot\left(1-\frac{\omega^2 \cdot R^2}{c^2}\right)} - 1\right] \quad (49)$$

- formulas for the projections P'_{x1i} and P'_{y1i} of the momentum P'_{1i} of the *i*-the point of string 3 on the axis of the Cartesian coordinates in the reference system O'*x'y'z'* at time t'_{1i}, the corresponding time t_{1i} in the reference system O*xyz*:

$$P'_{x1i} = \frac{v'_{x1i} \cdot \ m_{0n}}{\sqrt{1 - \frac{{v'_{1i}}^2}{c^2}}} = -\frac{m_{0n} \ \cdot \{[\omega \ \cdot R_i \ \ \cdot \sin(\omega \ \cdot \ t_{1i})] + V\}}{\sqrt{1 - \frac{V^2}{c^2}} \cdot \sqrt{1 - \frac{\omega^2 \ \cdot {R_i}^2}{c^2}}} \qquad (50)$$

$$P'_{y1i} = = \frac{v'_{y1i} \cdot \ m_{0n}}{\sqrt{1 - \frac{{v'_{1i}}^2}{c^2}}} = \frac{m_{0n} \ \cdot \ \omega \ \cdot R_i \ \cdot \cos(\omega \ \cdot \ t_{1i})}{\sqrt{1 - \frac{\omega^2 \ \cdot {R_i}^2}{c^2}}} \qquad (51)$$

$${P'_{1i}}^2 = {P'_{x1i}}^2 + {P'_{y1i}}^2 = {m_{0n}}^2 \cdot c^2 \cdot \left[\frac{\left\{1 + \frac{V \cdot \omega \ R_i \ \cdot \sin(\omega \cdot t_{1i})}{c^2}\right\}^2}{\left(1 - \frac{V^2}{c^2}\right) \cdot \left(1 - \frac{\omega^2 \ \cdot {R_i}^2}{c^2}\right)} - 1\right] \quad (52)$$

- formulas for the projections P'_{x2j} and P'_{y2j} of the momentum P'_{2j} of the *j*-the point of string 3 on the axis of the Cartesian coordinates in the reference system O'*x'y'z'* at time t'_{2j}, the corresponding time t_{2j} in the reference system O*xyz*:

$$P'_{x2j} = \frac{v'_{x2j} \cdot \ m_{0n}}{\sqrt{1 - \frac{{v'_{2j}}^2}{c^2}}} = \frac{m_{0n} \ \cdot \{[\omega \ \cdot R_j \ \ \cdot \sin(\omega \ \cdot \ t_{2j})] - V\}}{\sqrt{1 - \frac{V^2}{c^2}} \cdot \sqrt{1 - \frac{\omega^2 \ \cdot {R_j}^2}{c^2}}} \qquad (53)$$

$$P'_{y2j} = \frac{v'_{y2j} \cdot \ m_{0n}}{\sqrt{1 - \frac{{v'_{2j}}^2}{c^2}}} = -\frac{m_{0n} \ \cdot \ \omega \ \cdot R_j \ \ \cdot \cos(\omega \ \cdot \ t_{2j})}{\sqrt{1 - \frac{\omega^2 \ \cdot {R_j}^2}{c^2}}} \qquad (54)$$

$${P'_{2j}}^2 = {P'_{x2j}}^2 + {P'_{y2j}}^2 = {m_{0n}}^2 \cdot c^2 \cdot \left[\frac{\left\{1 - \frac{V \cdot \omega \cdot R_j \cdot \sin(\omega \cdot t_{2j})}{c^2}\right\}^2}{\left(1 - \frac{V^2}{c^2}\right) \cdot \left(1 - \frac{\omega^2 \ \cdot {R_j}^2}{c^2}\right)} - 1\right] \quad (55)$$

To determine the value of the momentum of the system of bodies 1 and 2 and string 3 in the reference system O'*x'y'z'* at time t' it is necessary, that the moments of times t'_1, t'_2, t'_{1i}, and t'_{2j} were are equal and are equal to t', ie:

$$t' = t'_1 = t'_2 = t'_{1i} = t'_{2j} = \frac{t_1 - \frac{V \cdot R \cdot \cos(\omega \cdot t_1)}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{t_2 + \frac{V \cdot R \cdot \cos(\omega \cdot t_2)}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} =$$

$$= \frac{t_{1i} - \frac{V \ \cdot R_i \ \cdot \cos(\omega \ \cdot \ t_{1i})}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{t_{2j} + \frac{V \ \cdot R_j \cdot \cos(\omega \ \cdot \ t_{2j})}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (56)$$

Given, that the reference system O'*x'y'z'* is inertial, can write the following formula for the projections P'_x and P'_y of the momentum P' of the closed mechanical system, consisting of bodies 1 and 2 and string 3,on the axis of the Cartesian coordinates at time t' in the reference system O'*x'y'z'*:

$$P'_x = P'_{x1} + P'_{x2} + \sum_{1}^{i=n} P'_{x1i} + \sum_{1}^{j=n} P'_{x2j} \quad (57)$$

$$P'_y = P'_{y1} + P'_{y2} + \sum_{1}^{i=n} P'_{y1i} + \sum_{1}^{j=n} P'_{y2j} \quad (58)$$

$$P'^2 = P'^{\,2}_x + P'^{\,2}_y \quad (59)$$

IV. THE RESULTS OF THE CALCULATION OF THE NUMERICAL EXAMPLE

To get a visual image depending of the momentum P' of the closed mechanical system, consisting of bodies 1 and 2 and string 3, from the time t' in the reference system O'$x'y'z$ we can consider a numerical example, by typing the following inputs:

$$\frac{V}{c} = 0{,}9 \quad (60)$$

$$\frac{\omega \cdot R}{c} = 0{,}8 \quad (61)$$

$$\frac{m_0}{M_0} = 0{,}1 \quad (62)$$

$$n = 10 \quad (63)$$

The calculation can be made as follows:

- setting the value of the time t' and using the formula (56) to determine the values of moments of times t_1, t_2, t_{1i} and t_{2j};

- continue to define the values of the projections and P'_{x1} and P'_{y1} of the momentum P'_1 of the body 1 (formulas (44) and (45)), and projections P'_{x2} and P'_{y2} of the momentum P'_2 of the body 2 (formulas (47) and (48)), and projections P'_{x1i} and P'_{y1i} of the momentum P'_{1i} of the *i*-the point of string 3 (formulas (50) and (51)), and projections P'_{x2j} and P'_{y2j} of the momentum P'_{2j} of the *j*-the point of string 3 (formulas (53) and (54)) for different values of moments of times t';

- then for different times t' to determine the values of the projections P'_x and P'_y of the momentum P' of the system of bodies 1 and 2 and string 3 (formulas (57) and (58)), of the absolute value $|P'|$ of the momentum P' using the formula (59), and of the angle α' between the direction of the momentum P' and the axis O'x', defined by the formula:

$$\alpha' = arctg\left(\frac{P'_y}{P'_x}\right) \quad (64)$$

The calculation results are shown in the graphs:

- a graph of the dependence of the value of projection P'_x of the momentum P' of the system of bodies 1 and 2 and string 3 on the value of time t' (with and without weight string 3), is shown in Fig.4;

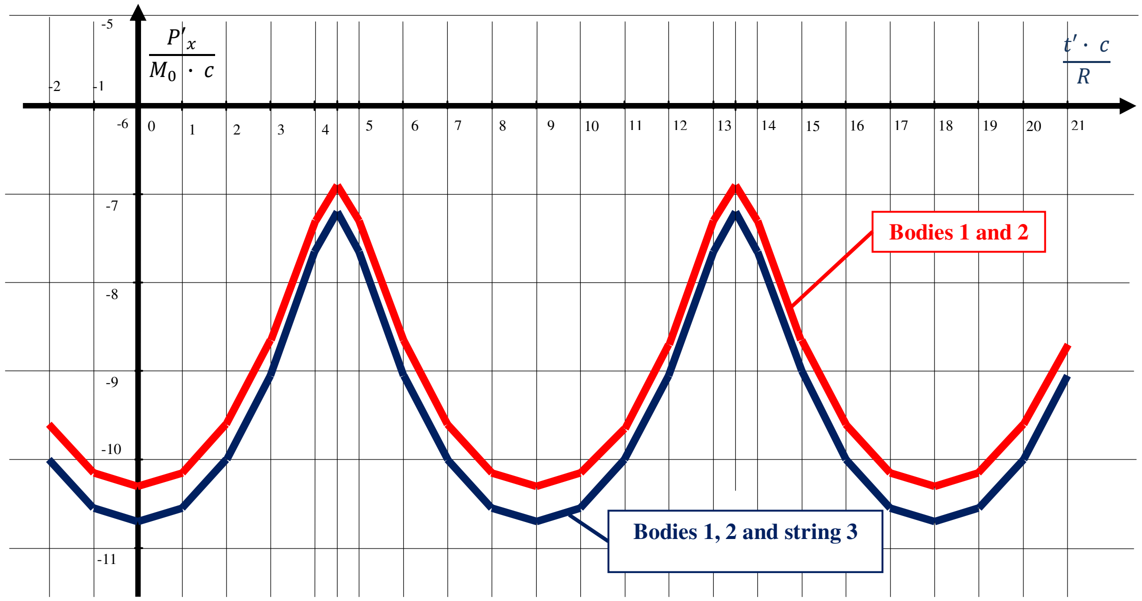

Fig.4

- a graph of the dependence of the value of projection P'_y of the momentum P' of the system of bodies 1 and 2 and string 3 on the value of time t' (with and without weight string 3), is shown in Fig.5;

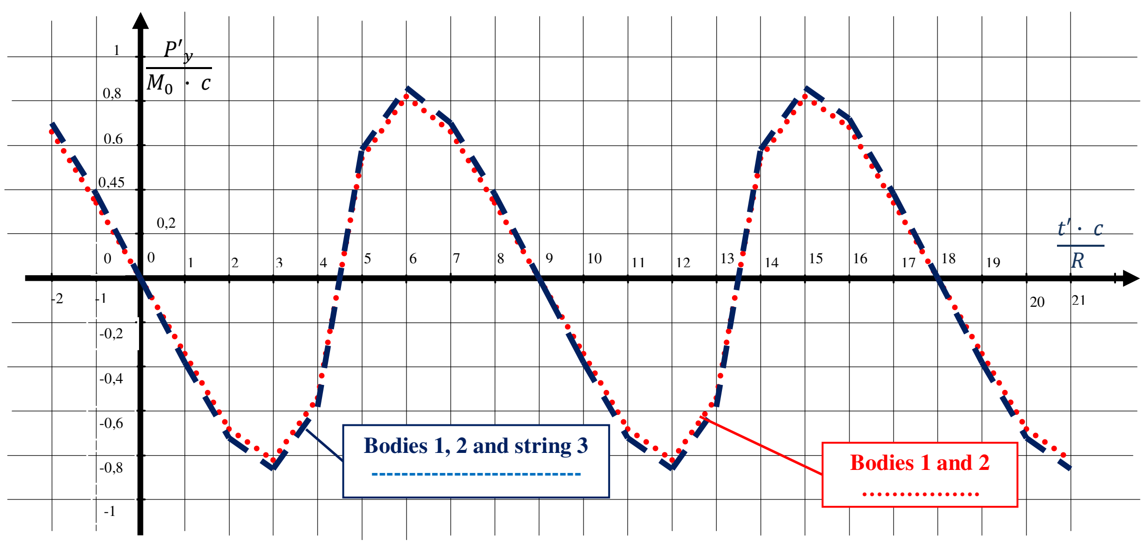

Fig.5

- a graph of the dependence of the absolute value $|P'|$ of the momentum P' of the system of bodies 1 and 2 and string 3 on the value of time t' (with and without weight string 3), is shown in Fig.6;

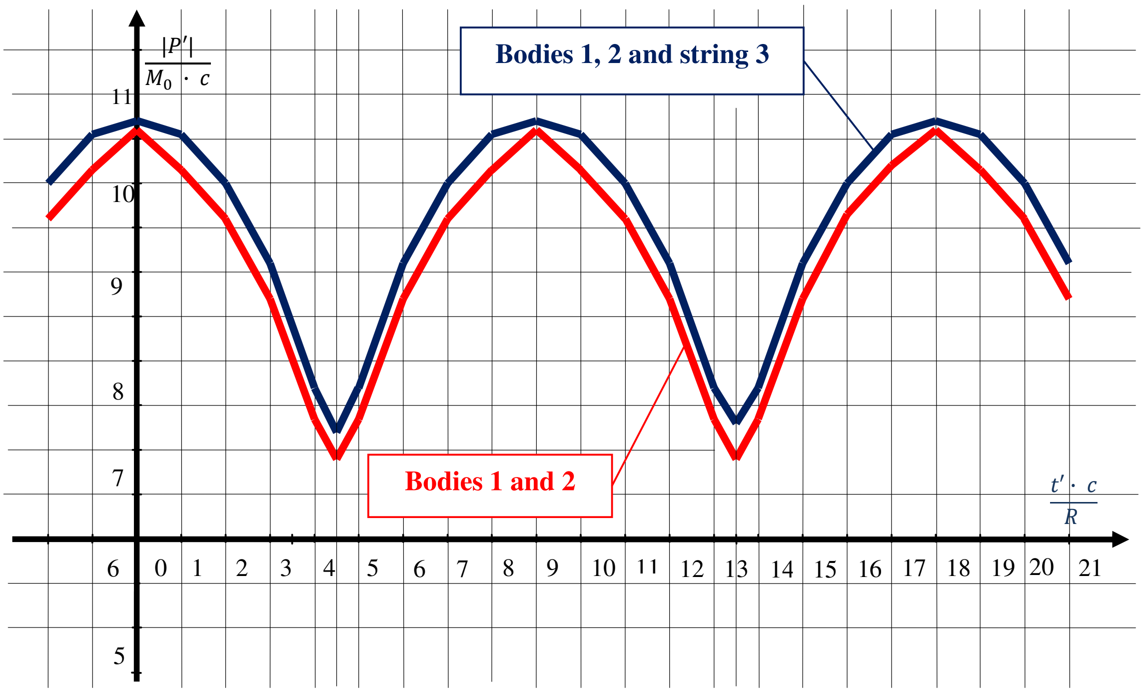

Fig.6

- a graph of the dependence of the value of the angle α' between the direction of the vector of the momentum *P'* of the system of bodies 1 and 2 and string 3 and the axis O'*x'* on the value of time *t'* (with and without weight string 3), is shown in Fig.7;

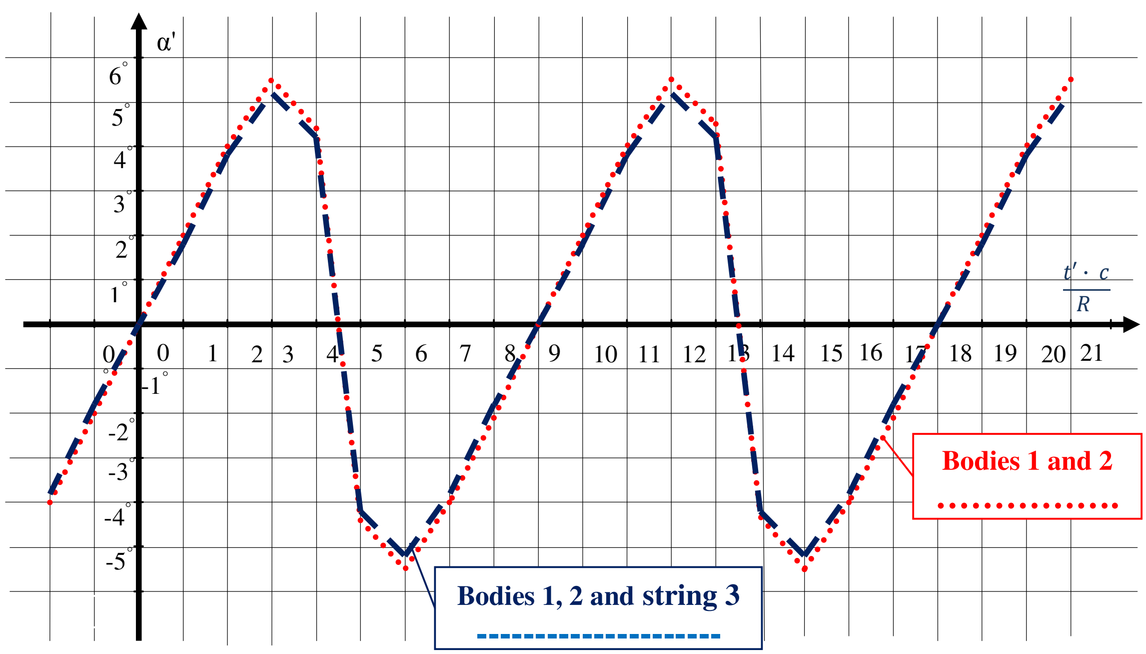

Fig.7

- graphs of the dependence of the values of projections P'_{x3} and P'_{y3} of the momentum P'_3 of the of the string 3 on the value of time *t'*, a graph of the dependence of the of the absolute value $|P'_3|$ of the momentum P'_3 of the of the string 3 on the value of time *t'*, is shown in Fig.8.

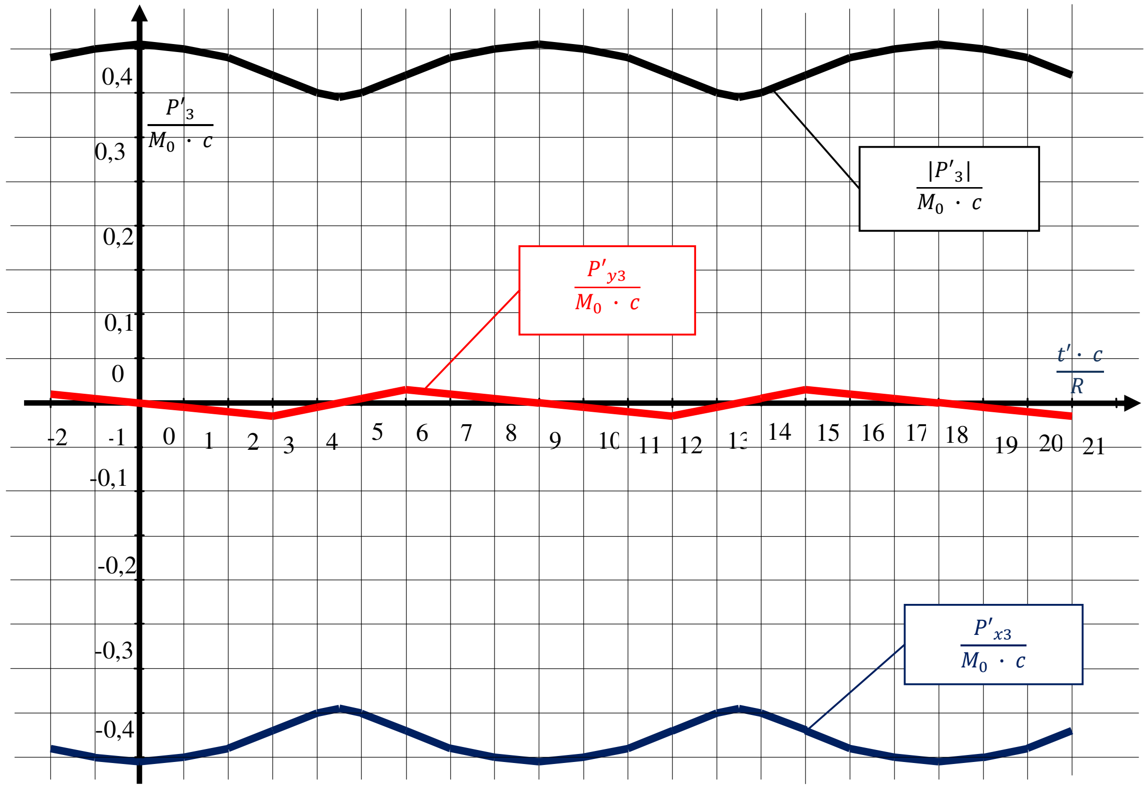

Fig. 8

As a result of the calculation, it was found, that the use of the special theory of relativity leads to the fact, that in the inertial reference system O'*x*'*y*'*z*' the closed mechanical system of bodies 1 and 2 and string 3 has variable in magnitude and direction of the vector of the momentum *P*' in the time *t*' (ie the momentum *P*' of this closed system is a function of time *t*'), which contradicts the law of conservation of momentum.

As a result, we can conclude, that in the inertial reference frame O'*x*'*y*'*z*' the application of the special theory of relativity in describing the motion of a closed mechanical system of bodies, considered in this example, leads to non-compliance with the law of conservation of momentum.

V. CONCLUSION

It can be concluded, that the use of the special theory of relativity in dealing with individual examples may lead to non-compliance with the law of conservation of momentum of a closed mechanical system in the inertial reference systems.

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