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Topic: special theory of relativity

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The special theory of relativity and the law of conservation of momentum

This article attempts to use the law of conservation of momentum of a closed system for determining the values of the constants in the two possible transformations of coordinates and times in inertial reference systems.

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1. The introduction

Special theory of relativity can be divided into relativistic kinematics and relativistic dynamics.

Relativistic kinematics, based on the symmetry of space and time [1], [2], [3], [4], [5], the principle of relativity and the principle of invariance of the speed of light [6], [7], [8], [9], [10], [11], [12], 13], allows to establish links between the coordinates and times (Lorentz transformation), speeds (the transformation of speeds) and accelerations (the transformation of accelerations) in the inertial reference systems.

The relativistic dynamics, based on the mandatory implementation of the laws of conservation of momentum, angular momentum and energy of a closed system [14], [15], [16], [17], [18], [19], [20], [21], [22], [23] in the inertial reference systems, determines the dependences of mass, momentum and energy point of the material body from its speed.

However, in the experiments [24] and the analysis of the results of observations [25], [26] were marked by inconsistency of the actual results the conclusions of the special theory of relativity.

To understand the reason that caused the deviation, can, by analogy with [4] consider the special theory of relativity in a general form with less severe conditions - without the use of the principle of invariance of the speed of light.

2. The special theory of relativity in the general form

Suppose that the space is homogeneous and isotropic and time is homogeneous (ie there is a symmetry of space and time).

In considering will only use the principle of relativity, asserts, that in any inertial reference systems all physical phenomena occur equally in the same conditions.

Assume that there are two inertial reference systems, shown in Fig.1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$, in which:

- similar the axis of the Cartesian coordinate systems $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$

are pairs parallel and equally directed;

- system $O_2x_2y_2z_2$ moves relative to the system $O_1x_1y_1z_1$ with constant speed V along the axis O_1x_1;

- in both systems as the start timing (t_1=0 and t_2=0) is selected when the origin O_1 and O_2 of these systems are identical.

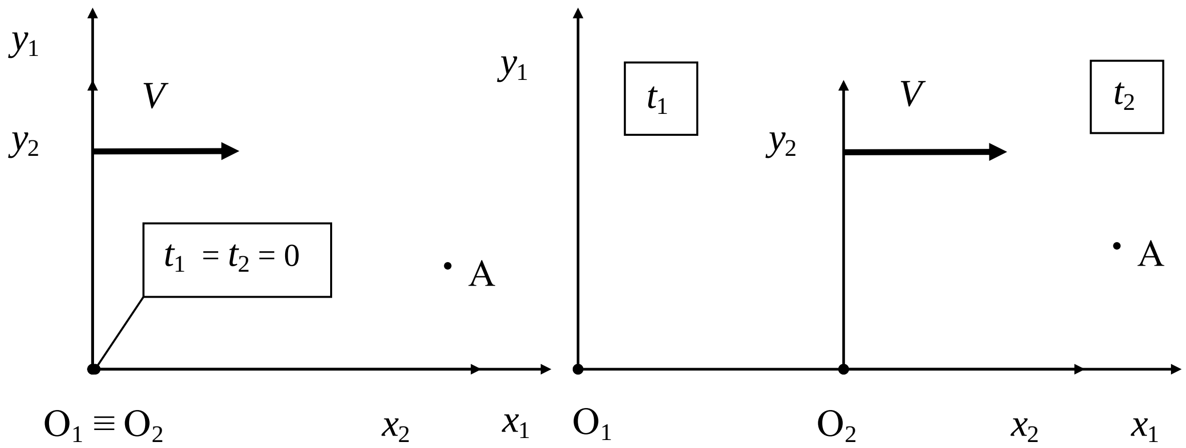

Fig.1

Using the principle of relativity and the symmetry of space and time, by analogy with [27], [8], [9], [10], [18], provides a link between the coordinates x_1, y_1, z_1 of point A at time t_1 in a stationary inertial reference system $O_1x_1y_1z_1$ and coordinates x_2, y_2, z_2 of the same point A in the mobile inertial reference system $O_2x_2y_2z_2$ at the time t_2, corresponding to time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$:

$$x_1 = \gamma_V \cdot [x_2 + (V \cdot t_2)] \quad (1)$$

$$x_2 = \gamma_V \cdot [x_1 - (V \cdot t_1)] \quad (2)$$

$$y_1 = y_2 \quad (3)$$

$$z_1 = z_2 \quad (4)$$

where: γ_V - coefficient of proportionality (the transition), which is presumably a function of speed V.

From formulas (1) and (2) we can write the dependence for times t_1 and t_2 :

$$t_1 = \frac{(\gamma_V^2 - 1) \cdot x_2}{\gamma_V \cdot V} + (\gamma_V \cdot t_2) \quad (5)$$

$$t_2 = \frac{(1 - \gamma_V^2) \cdot x_1}{\gamma_V \cdot V} + (\gamma_V \cdot t_1) \qquad (6)$$

Differentiating equations (1) - (6), we can obtain the relationship between the projections v_{x1}, v_{y1} and v_{z1} of the speed of a point on the axis of the Cartesian coordinates in time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$ and similar projections v_{x2}, v_{y2} and v_{z2} of the speed of the same point in the mobile inertial reference system $O_2x_2y_2z_2$ at time t_2, corresponding to time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$:

$$v_{x1} = \frac{v_{x2} + V}{\frac{(\gamma_V^2 - 1) \cdot v_{x2}}{\gamma_V^2 \cdot V} + 1} \qquad (7)$$

$$v_{x2} = \frac{v_{x1} - V}{\frac{(1 - \gamma_V^2) \cdot v_{x1}}{\gamma_V^2 \cdot V} + 1} \qquad (8)$$

$$v_{y1} = \frac{v_{y2}}{\frac{(\gamma_V^2 - 1) \cdot v_{x2}}{\gamma_V \cdot V} + \gamma_V} \qquad (9)$$

$$v_{y2} = \frac{v_{y1}}{\frac{(1 - \gamma_V^2) \cdot v_{x1}}{\gamma_V \cdot V} + \gamma_V} \qquad (10)$$

$$v_{z1} = \frac{v_{z2}}{\frac{(\gamma_V^2 - 1) \cdot v_{x2}}{\gamma_V \cdot V} + \gamma_V} \qquad (11)$$

$$v_{z2} = \frac{v_{z1}}{\frac{(1 - \gamma_V^2) \cdot v_{x1}}{\gamma_V \cdot v_{x1}} + \gamma_V} \qquad (12)$$

Differentiation of equations (7) - (12) and (5) - (6) will be written communication between the projections a_{x1}, a_{y1} and a_{z1} of the acceleration of a point on the axis of the Cartesian coordinates in time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$ and similar projections a_{x2}, a_{y2} and a_{z2} of the acceleration of the same point in the mobile inertial reference system $O_2x_2y_2z_2$ at time t_2 , corresponding to time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$:

$$a_{x1} = \frac{a_{x2}}{\gamma_V^3 \cdot \left[\frac{(\gamma_V^2 - 1) \cdot v_{x2}}{\gamma_V^2 \cdot V} + 1\right]^3} \qquad (13)$$

$$a_{x2} = \frac{a_{x1}}{\gamma_V^3 \cdot \left[\frac{(1 - \gamma_V^2) \cdot v_{x1}}{\gamma_V^2 \cdot V} + 1\right]^3} \qquad (14)$$

$$a_{y1} = \frac{\left\{a_{y2} \cdot \left[\frac{(\gamma_V^2 - 1) \cdot v_{x2}}{\gamma_V \cdot V} + \gamma_V\right]\right\} - \frac{(\gamma_V^2 - 1) \cdot a_{x2} \cdot v_{y2}}{\gamma_V \cdot V}}{\left[\frac{(\gamma_V^2 - 1) \cdot v_{x2}}{\gamma_V \cdot V} + \gamma_V\right]^3} \qquad (15)$$

$$a_{y2} = \frac{\left\{a_{y1} \cdot \left[\frac{(1 - \gamma_V^2) \cdot v_{x1}}{\gamma_V \cdot V} + \gamma_V\right]\right\} - \frac{(1 - \gamma_V^2) \cdot a_{x1} \cdot v_{y1}}{\gamma_V \cdot V}}{\left[\frac{(1 - \gamma_V^2) \cdot v_{x1}}{\gamma_V \cdot V} + \gamma_V\right]^3} \qquad (16)$$

$$a_{z1} = \frac{\left\{a_{z2} \cdot \left[\frac{(\gamma_V^2 - 1) \cdot v_{x2}}{\gamma_V \cdot V} + \gamma_V\right]\right\} - \frac{(\gamma_V^2 - 1) \cdot a_{x2} \cdot v_{z2}}{\gamma_V \cdot V}}{\left[\frac{(\gamma_V^2 - 1) \cdot v_{x2}}{\gamma_V \cdot V} + \gamma_V\right]^3} \qquad (17)$$

$$a_{z2} = \frac{\left\{a_{z1} \cdot \left[\frac{(1 - \gamma_V^2) \cdot v_{x1}}{\gamma_V \cdot V} + \gamma_V\right]\right\} - \frac{(1 - \gamma_V^2) \cdot a_{x1} \cdot v_{z1}}{\gamma_V \cdot V}}{\left[\frac{(1 - \gamma_V^2) \cdot v_{x1}}{\gamma_V \cdot V} + \gamma_V\right]^3} \qquad (18)$$

3. The equation of contact for the coefficients of proportionality

Consider three inertial reference systems, as shown in Fig.2, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$ and $O_3x_3y_3z_3$, in which:

- similar the axis of the Cartesian coordinate systems $O_1x_1y_1z_1$, $O_2x_2y_2z_2$ and $O_3x_3y_3z_3$ are parallel to the three and equally directed;

- system $O_2x_2y_2z_2$ moves relative to the system $O_1x_1y_1z_1$ with constant speed V_2 along the axis O_1x_1;

- system $O_3x_3y_3z_3$ moves relative to the system $O_1x_1y_1z_1$ with constant speed V_3 along the axis O_1x_1;

- in these three systems as the start timing (t_1=0 , t_2=0 and t_3=0) is selected, when their origin O_1, O_2 and O_3 are the same.

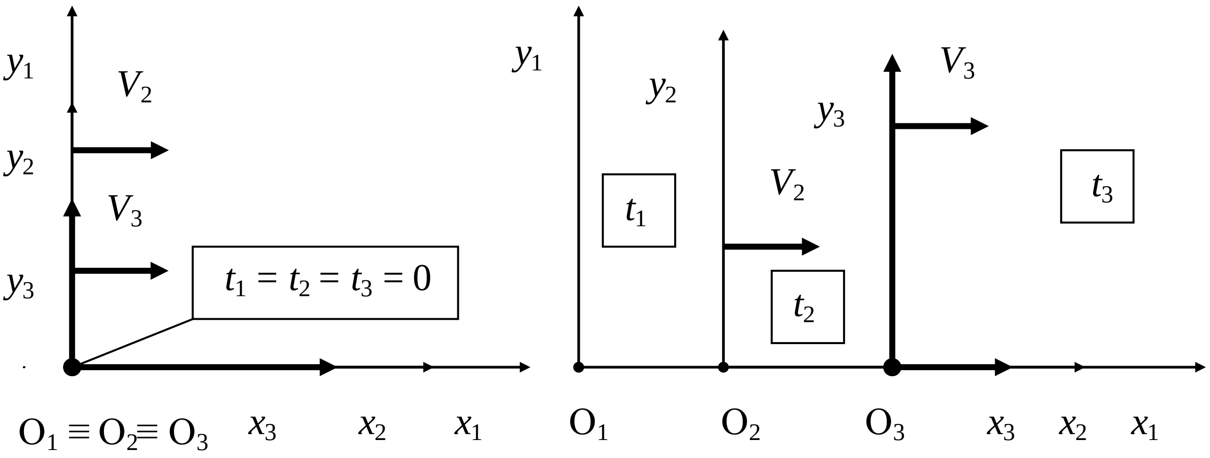

Fig.2

Based on the formula (8), we can determine the value of the speed V_{23} of the point O_3 relative to the point O_2:

$$V_{23} = \frac{V_3 - V_2}{\frac{(1 - \gamma_{V2}^2) \cdot V_3}{\gamma_{V2}^2 \cdot V_2} + 1} \tag{19}$$

and the value of the speed V_{32} of the point O_2 relative to the point O_3:

$$V_{32} = \frac{V_2 - V_3}{\frac{(1 - \gamma_{V3}^2) \cdot V_2}{\gamma_{V3}^2 \cdot V_3} + 1} \tag{20}$$

where: γ_{V2} and γ_{V3} - coefficients of proportionality for inertial reference systems moving relative to a stationary reference system at the speed V_2 and V_3, respectively.

Using the principle of relativity, by which point O_3 will be removed on the point O_2 at the speed equal in magnitude and oppositely directed of the speed, with which the point O_2 is removed relative to the point O_3, ie:

$$V_{32} = -V_{23} \tag{21}$$

Substituting equation (21) into formulas (19) and (20), we obtain:

$$\frac{(1 - \gamma_{V2}^2) \cdot V_3}{\gamma_{V2}^2 \cdot V_2} + 1 = \frac{(1 - \gamma_{V3}^2) \cdot V_2}{\gamma_{V3}^2 \cdot V_3} + 1 \tag{22}$$

From equation (22), it follows that:

$$\frac{\gamma_{V2}^2 - 1}{\gamma_{V2}^2 \cdot V_2^2} = \frac{\gamma_{V3}^2 - 1}{\gamma_{V3}^2 \cdot V_3^2} \qquad (23)$$

Since the values of the coefficients of proportionality γ_{V2} and γ_{V3} not depend on each other and depend only on the values of the speeds V_2 and V_3, respectively, and the speeds V_2 and V_3 were set arbitrarily (and do not depend on each other), then we can say that:

$$\frac{\gamma_{V2}^2 - 1}{\gamma_{V2}^2 \cdot V_2^2} = \frac{\gamma_{V3}^2 - 1}{\gamma_{V3}^2 \cdot V_3^2} = K = Const \qquad (24)$$

ie obtained in general terms that:

$$\frac{\gamma_V^2 - 1}{\gamma_V^2 \cdot V^2} = K = Const \qquad (25)$$

where: K - constant, irrespective of the magnitude of the speed V and the coefficient of proportionality γ_V.

As seen from formula (25), depending on the value of the coefficient of proportionality γ_V the constant K may have the following meanings:

- with $\gamma_V = 1$ the constant K will be equal to 0;

- if the coefficient of proportionality $\gamma_V > 1$, then the constant K will be positive, ie $K > 0$;

- if the coefficient of proportionality $0 < \gamma_V < 1$, then the constant K will have a negative value, ie $K < 0$.

From equation (25) can obtain a formula for the coefficient of proportionality γ_V:

$$\gamma_V^2 = \frac{1}{1 - (K \cdot V^2)} \qquad (26)$$

By analogy with the special theory of relativity, we assume that:

- for values of the coefficient of proportionality $\gamma_V > 1$ the constant K is equal to:

$$K = \frac{1}{c_1^2} \qquad (27)$$

- for values of the coefficient of proportionality $0 < \gamma_V < 1$ the constant K is equal to:

$$K = -\frac{1}{c_2^2} \qquad (28)$$

where: c_1 and c_2 - real constants.

4. Determination of specific speed

Suppose that in a stationary reference system $O_1x_1y_1z_1$ there exists a value v_{xkr} of the speed projection v_{x1} of point A, shown in Fig. 1, which in the mobile reference system $O_2x_2y_2z_2$ would be consistent value of the speed projection v_{x2} of the same point A, equal v_{xkr} , ie .:

$$v_{x1} = v_{x2} = v_{xkr} \qquad (29)$$

We call v_{xkr} specific speed.

Substituting (29) into formula (7) or (8), we obtain the dependence of the specific speed v_{xkr} on the magnitude of the speed V and the coefficient of proportionality γ_V :

$$v_{xkr}^2 = \frac{\gamma_V^2 \cdot V^2}{\gamma_V^2 - 1} \qquad (30)$$

As seen from formula (30):

- for values of the coefficient of proportionality γ_V, which are in the range $\gamma_V > 1$, equity of the projections v_{x1} and v_{x2} of speeds is possible, because when $\gamma_V > 1$ the specific speed v_{xkr} will have real meaning;

- for values of the coefficient of proportionality γ_V , which are in the range $0 < \gamma_V < 1$, the equality of the projections v_{x1} and v_{x2} of speeds is not possible, ie value v_{x1} can never be equal to the value v_{x2} , because when $0 < \gamma_V < 1$ the specific speed v_{xkr} will have an imaginary value.

From formula (30) can be obtained dependence of the coefficient of proportionality γ_V on the magnitude of the speed V:

$$\gamma_V^2 = \frac{1}{1 - \frac{V^2}{v_{xkr}^2}} \qquad (31)$$

If we return to formula (26) and compare it with the formula (31), we can see that:

$$K = \frac{1}{v_{xkr}^2} \qquad (32)$$

Based on the fact that the constant *K* is a constant, it may be noted that v_{xkr}^2 will be constant, independent of the values of the speed *V* and the coefficient of proportionality γ_V.

5. Basic kinematic equations for the values of the coefficient of proportionality γ_V in the range $\gamma_V > 1$ and $0 < \gamma_V < 1$

Using formula (31) taking into account equation (27) for the coefficient of proportionality γ_V , which has the values $\gamma_V > 1$, which is denoted as $\gamma_{V>}$, we can write:

$$\gamma_{V>}^2 = \frac{1}{1 - \frac{V^2}{c_1^2}} \qquad (33)$$

And from the formula (31) taking into account equation (28) for the coefficient of proportionality γ_V , having the values $0 < \gamma_V < 1$, which is denoted as $\gamma_{V<}$, we can get:

$$\gamma_{V<}^2 = \frac{1}{1 + \frac{V^2}{c_2^2}} \qquad (34)$$

Equation, similar to formula (34), was obtained Y.P. Terletsky [4] and discarded them without a theoretical justification as contrary to experience.

Substituting formulas (33) and (34) in equation (1) - (14), we obtain two systems of equations, which are located opposite each other for comparison, and the sign «>» indicates that this is the case when $\gamma_V > 1$, and sign «<» - for the case when $0 < \gamma_V < 1$:

$$x_{1>} = \frac{x_{2>} + (V \cdot t_{2>})}{\sqrt{1 - \frac{V^2}{c_1^2}}} \qquad (35)$$

$$x_{1<} = \frac{x_{2<} + (V \cdot t_{2<})}{\sqrt{1 + \frac{V^2}{c_2^2}}} \qquad (49)$$

$$x_{2>} = \frac{x_{1>} - (V \cdot t_{1>})}{\sqrt{1 - \frac{V^2}{c_1^2}}} \quad (36)$$

$$y_{1>} = y_{2>} \quad (37)$$

$$z_{1>} = z_{2>} \quad (38)$$

$$t_{1>} = \frac{t_{2>} + \frac{V \cdot x_{2>}}{c_1^2}}{\sqrt{1 - \frac{V^2}{c_1^2}}} \quad (39)$$

$$t_{2>} = \frac{t_{1>} - \frac{V \cdot x_{1>}}{c_1^2}}{\sqrt{1 - \frac{V^2}{c_1^2}}} \quad (40)$$

$$v_{x1>} = \frac{v_{x2>} + V}{1 + \frac{V \cdot v_{x2>}}{c_1^2}} \quad (41)$$

$$v_{x2>} = \frac{v_{x1>} - V}{1 - \frac{V \cdot v_{x1>}}{c_1^2}} \quad (42)$$

$$v_{y1>} = \frac{v_{y2>} \cdot \sqrt{1 - \frac{V^2}{c_1^2}}}{1 + \frac{V \cdot v_{x2>}}{c_1^2}} \quad (43)$$

$$v_{y2>} = \frac{v_{y1>} \cdot \sqrt{1 - \frac{V^2}{c_1^2}}}{1 - \frac{V \cdot v_{x1>}}{c_1^2}} \quad (44)$$

$$v_{z1>} = \frac{v_{z2>} \cdot \sqrt{1 - \frac{V^2}{c_1^2}}}{1 + \frac{V \cdot v_{x2>}}{c_1^2}} \quad (45)$$

$$x_{2<} = \frac{x_{1<} - (V \cdot t_{1<})}{\sqrt{1 + \frac{V^2}{c_2^2}}} \quad (50)$$

$$y_{1<} = y_{2<} \quad (51)$$

$$z_{1<} = z_{2<} \quad (52)$$

$$t_{1<} = \frac{t_{2<} - \frac{V \cdot x_{2<}}{c_2^2}}{\sqrt{1 + \frac{V^2}{c_2^2}}} \quad (53)$$

$$t_{2<} = \frac{t_{1<} + \frac{V \cdot x_{1<}}{c_2^2}}{\sqrt{1 + \frac{V^2}{c_2^2}}} \quad (54)$$

$$v_{x1<} = \frac{v_{x2<} + V}{1 - \frac{V \cdot v_{x2<}}{c_2^2}} \quad (55)$$

$$v_{x2<} = \frac{v_{x1<} - V}{1 + \frac{V \cdot v_{x1<}}{c_2^2}} \quad (56)$$

$$v_{y1<} = \frac{v_{y2<} \cdot \sqrt{1 + \frac{V^2}{c_2^2}}}{1 - \frac{V \cdot v_{x2<}}{c_2^2}} \quad (57)$$

$$v_{y2<} = \frac{v_{y1<} \cdot \sqrt{1 + \frac{V^2}{c_2^2}}}{1 + \frac{V \cdot v_{x1<}}{c_2^2}} \quad (58)$$

$$v_{z1<} = \frac{v_{z2<} \cdot \sqrt{1 + \frac{V^2}{c_2^2}}}{1 - \frac{V \cdot v_{x2<}}{c_2^2}} \quad (59)$$

$$v_{z2>} = \frac{v_{z1>} \cdot \sqrt{1 - \frac{V^2}{c_1^2}}}{1 - \frac{V \cdot v_{x1>}}{c_1^2}} \quad (46)$$

$$a_{x1>} = \frac{a_{x2>} \cdot \left(\sqrt{1 - \frac{V^2}{c_1^2}}\right)^3}{\left(1 + \frac{V \cdot v_{x2>}}{c_1^2}\right)^3} \quad (47)$$

$$a_{x2>} = \frac{a_{x1>} \cdot \left(\sqrt{1 - \frac{V^2}{c_1^2}}\right)^3}{\left(1 - \frac{V \cdot v_{x1>}}{c_1^2}\right)^3} \quad (48)$$

$$v_{z2<} = \frac{v_{z1<} \cdot \sqrt{1 + \frac{V^2}{c_2^2}}}{1 + \frac{V \cdot v_{x1<}}{c_2^2}} \quad (60)$$

$$a_{x1<} = \frac{a_{x2<} \cdot \left(\sqrt{1 + \frac{V^2}{c_2^2}}\right)^3}{\left(1 - \frac{V \cdot v_{x2<}}{c_2^2}\right)^3} \quad (61)$$

$$a_{x2<} = \frac{a_{x1<} \cdot \left(\sqrt{1 + \frac{V^2}{c_2^2}}\right)^3}{\left(1 + \frac{V \cdot v_{x1<}}{c_2^2}\right)^3} \quad (62)$$

For convenience of comparison the above formulas, you can use the following graphs:

- graph, shown in Fig.3, of the length of the segment Δx_1 in the stationary reference system $O_1x_1y_1z_1$, which in the mobile inertial reference system $O_2x_2y_2z_2$ corresponds to the segment Δx_2 , having fixed ends, on the speed V:

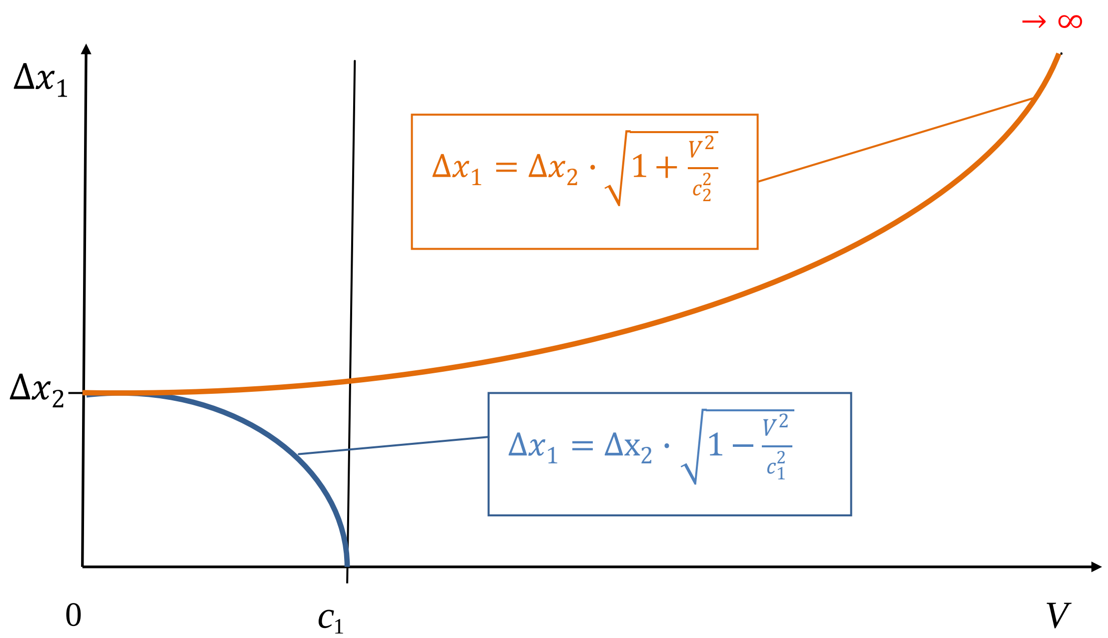

Fig.3

- graph, shown in Fig.4, of the time interval Δt_1 between two events in the stationary reference system $O_1x_1y_1z_1$, which in the mobile inertial reference system $O_2x_2y_2z_2$ occurred in the time interval Δt_2 in the same point on the speed V:

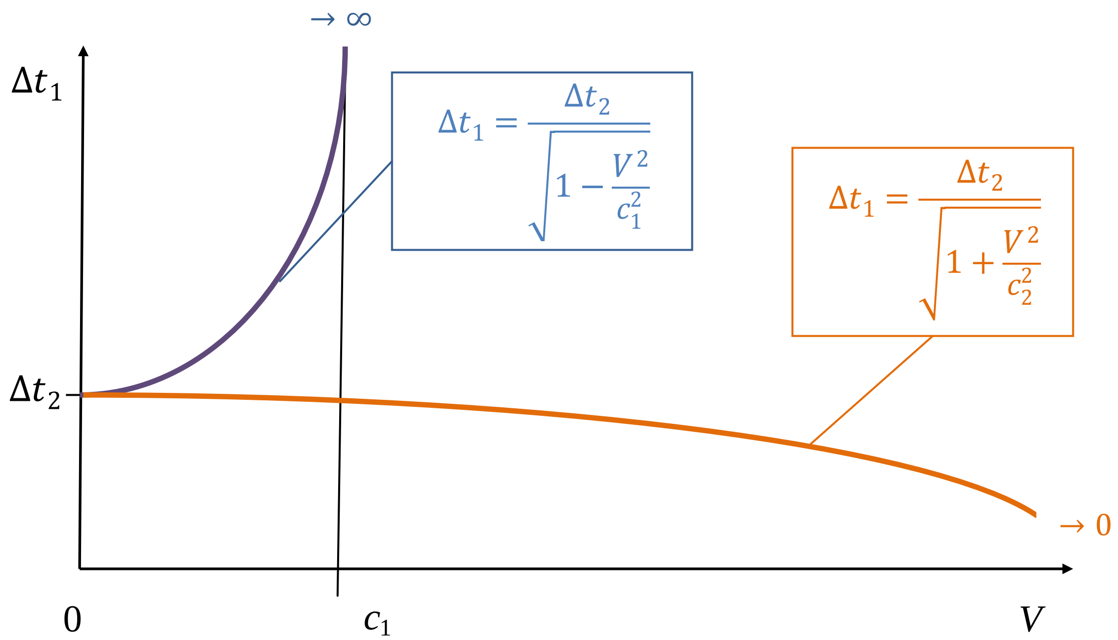

Fig.4

- graph, shown in Fig.5, of the relationship between the projection v_{x2} of the speed of the point in the mobile inertial reference system $O_2x_2y_2z_2$ and the projection v_{x1} of the speed of this point in the stationary reference system $O_1x_1y_1z_1$ (at constant speed V):

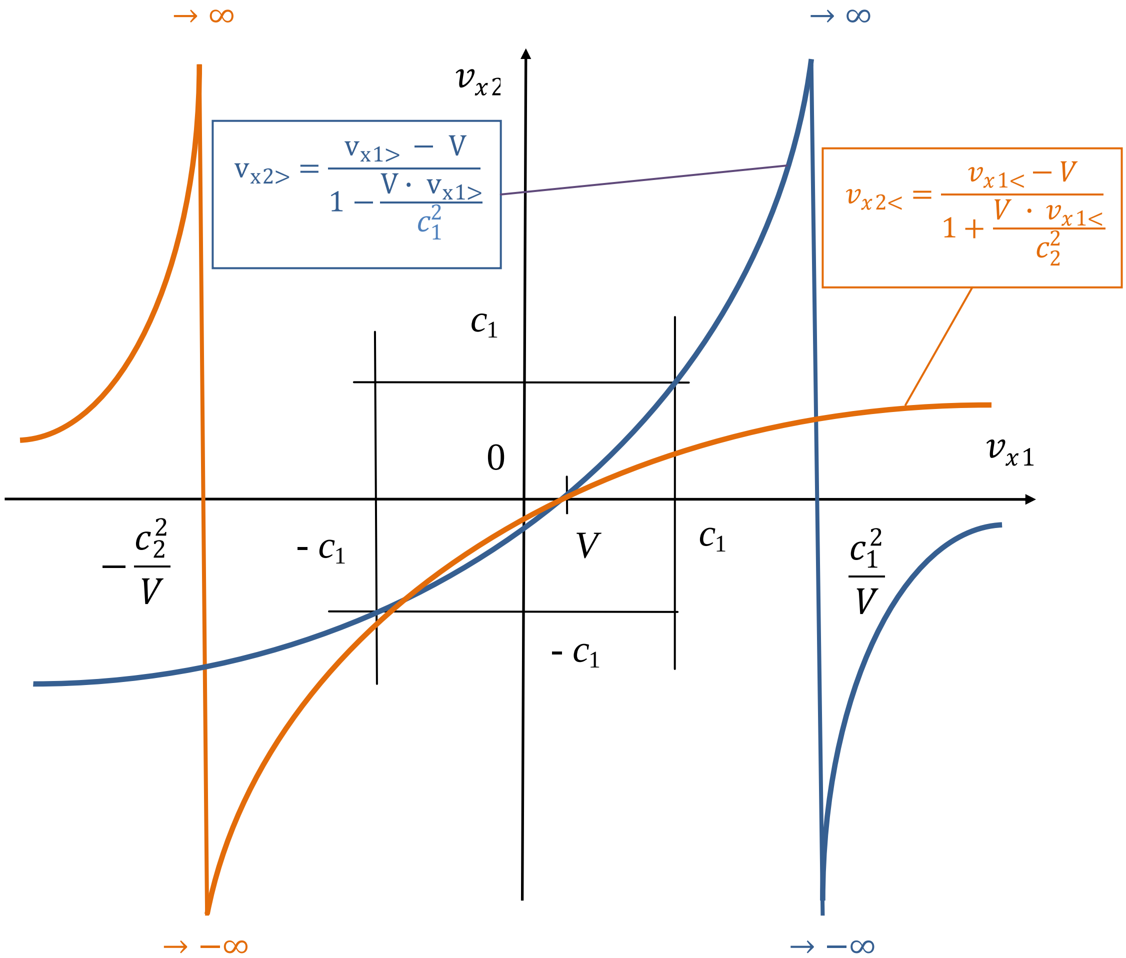

Fig.5

- graph, shown in Fig.6, of the relationship between the projection v_{x1} of the speed of the point in the stationary reference system $O_1x_1y_1z_1$ and the projection v_{x2} of the speed of this point in the mobile inertial reference system $O_2x_2y_2z_2$ (at constant speed V):

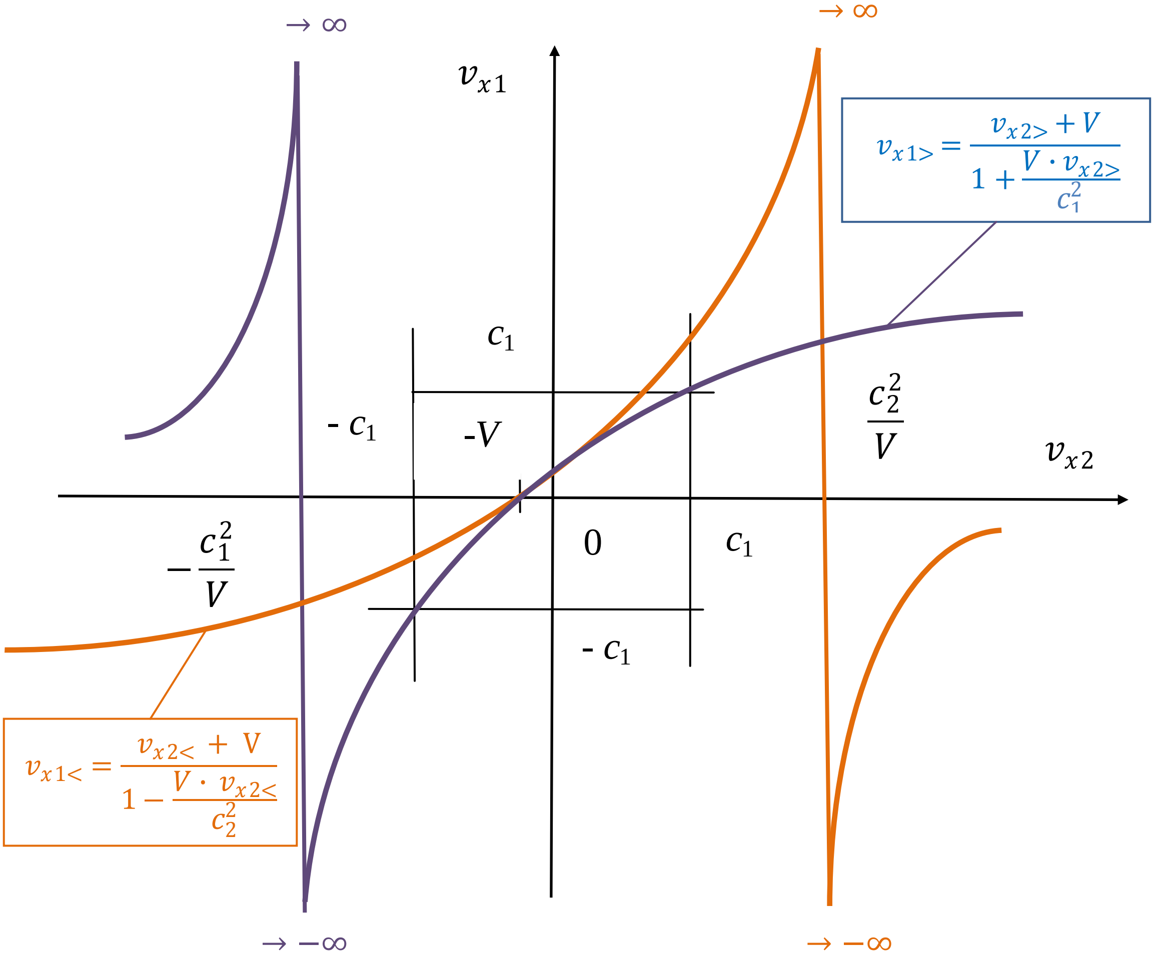

Fig.6

- graph, shown in Fig.7, of the relationship between the projection a_{x2} of the acceleration of the point in the mobile inertial reference system $O_2x_2y_2z_2$ and the projection v_{x1} of the speed of this point in the stationary reference system $O_1x_1y_1z_1$ (at constant speed V and constant projection a_{x1} of the acceleration):

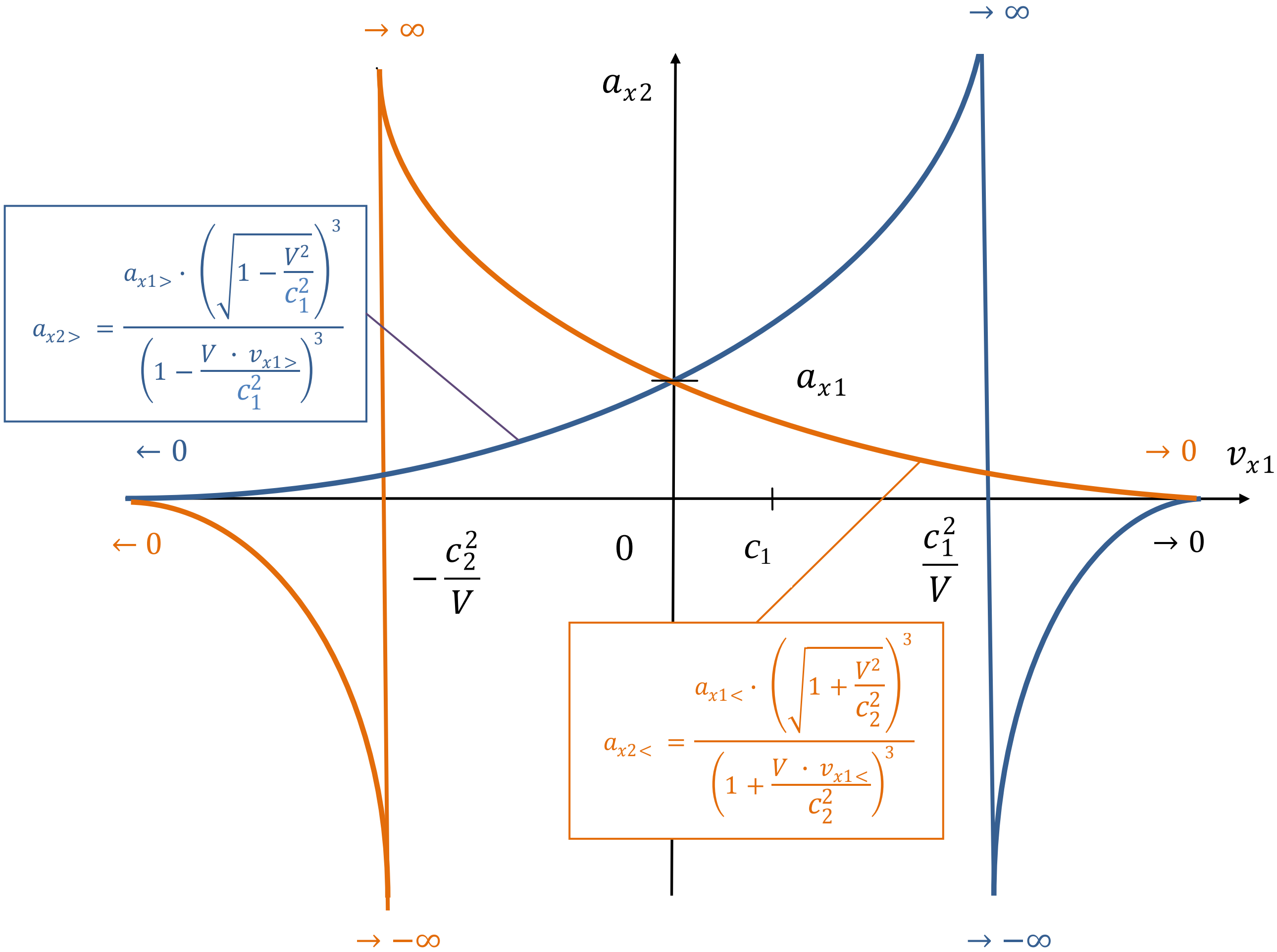

Fig.7

- graph, shown in Fig.8, of the relationship between the projection a_{x1} of the acceleration of the point in the stationary reference system $O_1x_1y_1z_1$ and the projection v_{x2} of the speed of this point in the mobile inertial reference system $O_2x_2y_2z_2$ (at constant speed V and constant projection a_{x2} of the acceleration):

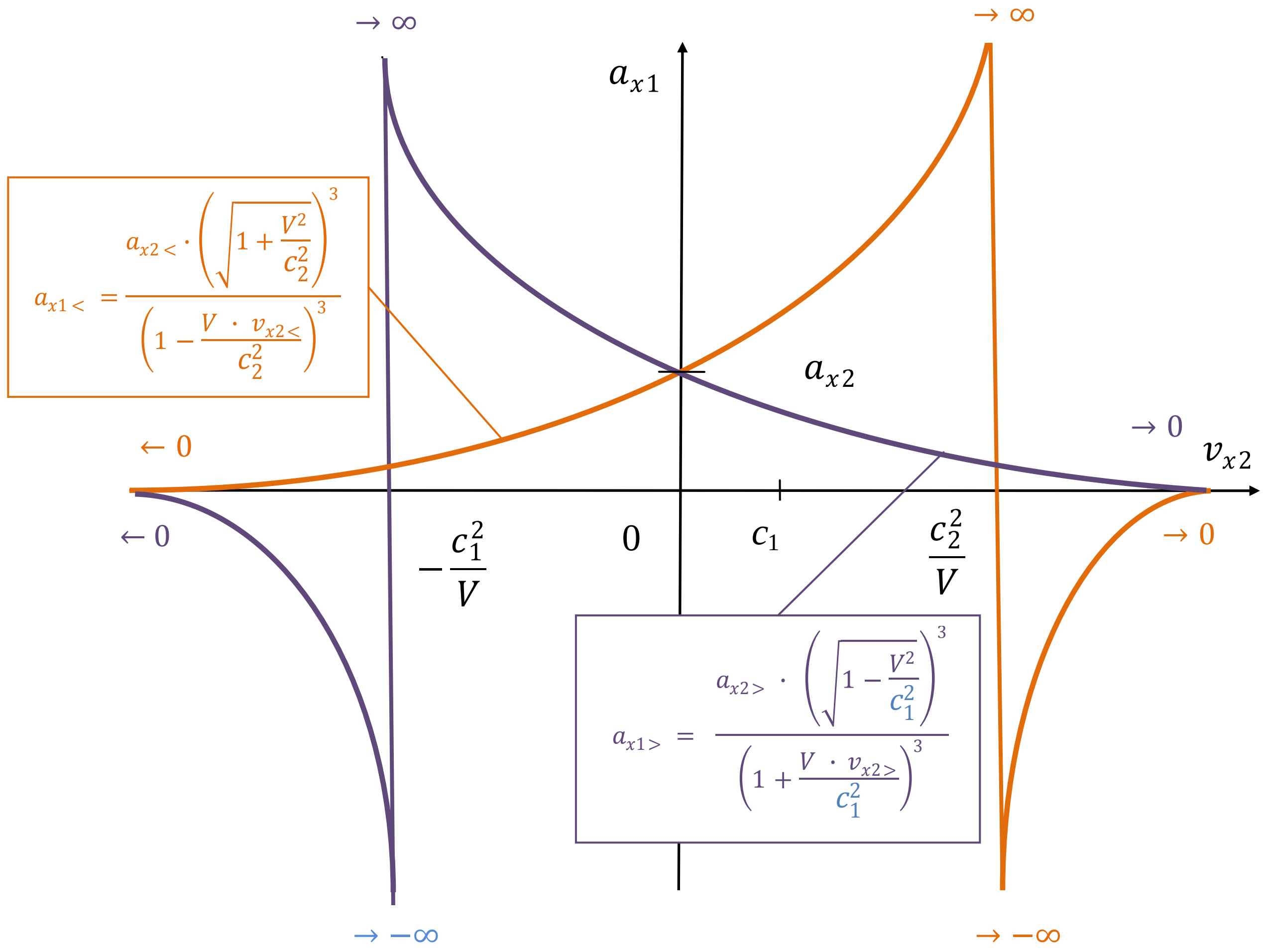

Fig.8

Also be noted that all physical processes occur in the four-dimensional space-time, whose geometry - is pseudoeuclidean and determined invariant $J = (c_1^2 \cdot t^2) - x^2 - y^2 - z^2$ [12] for the case when the coefficient of proportionality $\gamma_V > 1$, and the invariant $J = (c_2^2 \cdot t^2) + x^2 + y^2 + z^2$ for the case when the coefficient of proportionality $0 < \gamma_V < 1$.

6. Dependences of mass, momentum and kinetic energy of the moving body from its speed

Based on the mandatory implementation of the laws of conservation of momentum and energy of a closed mechanical system, the dependence of mass $M(v)$ of a moving body on the speed v can be obtained using the Lagrangian [1], [8], [9], [12], [14], [23], when considering the perfectly elastic or perfectly plastic collision [15], [16], [17], [18], [19], [20], [21], or just intuitively [13],

[22].

Also, the dependence of mass $M(v)$ of the moving the body on the speed v can be obtained by selecting the function of this dependence in the equations, written for two inertial reference systems, based on the laws of conservation of momentum and energy of a closed mechanical system consisting of two bodies facing perfectly elastic direct central collision, bearing short-term nature, with different positions of the system of bodies in space [28].

Summarizing the results of the findings [8], [13], [22], [28], the dependence of mass $M(v)$ of the moving body, having a rest mass M_o , on the speed v is as follows:

$$M(v) = M_o \cdot \gamma_v \qquad (63)$$

where: γ_v - coefficient of proportionality with the speed V, equal to v.

Knowing the relationship [1], [28] between the mass of the moving body and its momentum $P(v)$ and the kinetic energy $E_{kin}(v)$, we can write:

$$P(v) = M_o \cdot \gamma_v \cdot v \qquad (64)$$

$$E_{kin}(v) = \frac{M_0 \cdot \gamma_v^2 \cdot v^2}{\gamma_v + 1} \qquad (65)$$

Check the correctness of the choice of formulas (63) - (65) can be in the following example 1.

Assume that there are two inertial reference systems, similar system of reference, shown in Fig.1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$, which moves with speed V parallel to the axis O_1x_1 on the system $O_1x_1y_1z_1$.

Suppose that there is a closed mechanical system, consisting of a body 1 and body 2 (as shown in Fig.9), with a masses at rest, equal M_{o1} and M_{o2}, respectively.

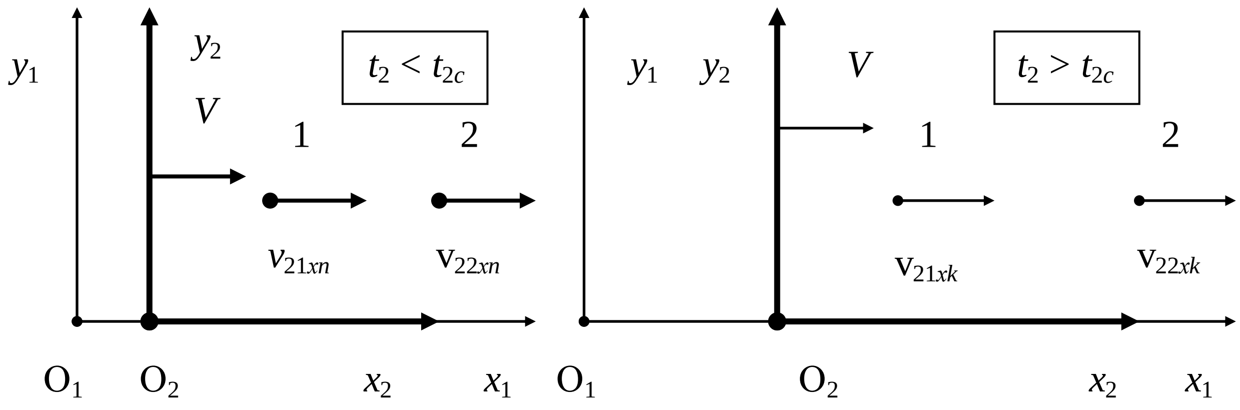

Fig.9

In the mobile reference system $O_2x_2y_2z_2$ to a certain point in time t_{2c} body 1 and body 2 moving parallel to the axis O_2x_2 on one line with constant speeds v_{21xn} and v_{22xn} respectively, ie to the time, smaller t_{2c}, body 1 had the momentum $P_{21\,xn}$ and the kinetic energy $E_{kin\,21\,xn}$, and the body 2 had the momentum $P_{22\,xn}$ and the kinetic energy $E_{kin\,22\,xn}$.

At some point in time t_{2c} between bodies 1 and 2, there was a direct central perfectly elastic collision.

Then, after the collision, at the time, more t_{2c} , bodies 1 and 2 are moving parallel to the axis O_2x_2 on one line with constant speeds v_{21xk} and v_{22xk} respectively, ie at the time, greater t_{2c}, body 1 had the momentum $P_{21\,xk}$ and the kinetic energy $E_{kin\,21\,xk}$, and the body 2 had the momentum $P_{22\,xk}$ and the kinetic energy $E_{kin\,22\,xk}$.

In the stationary reference system $O_1x_1y_1z_1$ collision between bodies 1 and 2 occurred at the time t_{1c}, corresponding to time t_{2c} in the mobile reference system $O_2x_2y_2z_2$.

In the stationary reference system $O_1x_1y_1z_1$ to a certain point in time t_{1c} body 1 and body 2 moving parallel to the axis O_1x_1 on one line with constant speeds v_{11xn} and v_{12xn} , respectively, ie to the time, smaller t_{1c} , body 1 had the momentum $P_{11\,xn}$ and the kinetic energy $E_{kin\,11\,xn}$, and the body 2 had the momentum $P_{12\,xn}$ and the kinetic energy $E_{kin\,12\,xn}$.

Then, after the collision, at the time, more t_{1c} , bodies 1 and 2 are moving parallel to the axis O_1x_1 on one line with constant speeds v_{11xk} and v_{12xk} respectively, ie at the time, greater t_{1c} , body 1 had the momentum $P_{11\,xk}$ and the kinetic energy $E_{kin\,11\,xk}$, and the body 2 had the momentum $P_{12\,xk}$ and the kinetic energy $E_{kin\,12\,xk}$.

Given that:

- has the symmetry of space and time,
- body 1 and body 2 form the closed mechanical system,
- between bodies 1 and 2 there was a direct central collision,
- collision between bodies 1 and 2 was of an elastic nature,

we can write the conservation laws of momentum and mechanical energy for the closed mechanical system, consisting of bodies 1 and 2, considering the moments before and after the collision:

in the stationary reference system $O_1x_1y_1z_1$:

$$P_{11\,xn} + P_{12\,xn} = P_{11\,xk} + P_{12\,xk} \qquad (66)$$

$$E_{kin\,11\,xn} + E_{kin\,12\,xn} = E_{kin\,11\,xk} + E_{kin\,12\,xk} \qquad (67)$$

in the mobile reference system $O_2x_2y_2z_2$:

$$P_{21\,xn} + P_{22\,xn} = P_{21\,xk} + P_{22\,xk} \qquad (68)$$

$$E_{kin\,21\,xn} + E_{kin\,22\,xn} = E_{kin\,21\,xk} + E_{kin\,22\,xk} \qquad (69)$$

Should also be noted that the speeds v_{11xn} and v_{21xn} , v_{12xn} and v_{22xn} , v_{11xk} and v_{21xk} , v_{12xk} and v_{22xk} are interconnected through a conversion speeds (7):

$$v_{x11\,xn} = \frac{v_{x21\,xn} + V}{\frac{(\gamma_V^2 - 1) \cdot v_{x2xn}}{\gamma_V^2 \cdot V} + 1} \qquad (70)$$

etc.

Now, by setting the initial data, we can hold the estimated test of choice of dependences (63) - (65) mass, momentum and kinetic energy of the moving body for the case, when the coefficients of proportionality γ_V and γ_v are in the ranges $\gamma_V > 1$ and $\gamma_v > 1$, respectively.

Suppose that: $M_{o1} = 1$, $M_{o2} = 0{,}5$, $V / c_1 = 0{,}5$, $v_{21xn} / c_1 = 0{,}9$, $v_{22xn} / c_1 = 0{,}6$.

Then, for this example 1, with coefficients of proportionality γ_V and γ_v, whose values may be in the ranges $\gamma_V > 1$ and $\gamma_v > 1$, the numerical calculations yield the following results in Tab.1 for the mobile reference system $O_2x_2y_2z_2$ and Tab.2 for the stationary reference system $O_1x_1y_1z_1$.

Tab.1

Ranges $\gamma_V > 1$ and $\gamma_v > 1$. The mobile reference system $O_2x_2y_2z_2$.

Object	Period time	Value	Digit of value
Body 1	Before collision	speed v_{21xn} / c_1	0,9
		mass M_{21n}	2,294157338706
		momentum P_{21xn} / c_1	2,064741604835
		kinetic energy $E_{kin\,21\,xn} / c_1^2$	1,294157338706
	After collision	speed v_{11xk} / c_1	0,7360143377
		mass M_{21k}	1,477179174242
		momentum P_{21xk} / c_1	1,087225051595
		kinetic energy $E_{kin\,21\,xk} / c_1^2$	0,477179174242
Body 2	Before collision	speed v_{22xn} / c_1	0,6
		mass M_{22n}	0,625
		momentum P_{22xn} / c_1	0,375
		kinetic energy $E_{kin\,22\,xn} / c_1^2$	0,125
	After collision	speed v_{22xk} / c_1	0,937959108239
		mass M_{22k}	1,441978164463
		momentum P_{22xk} / c_1	1,35251655324
		kinetic energy $E_{kin\,22\,xk} / c_1^2$	0,941978164463
The system of bodies 1 and 2	Before collision	mass $(M_{21n} + M_{22n})$	2,919157338706
		momentum $(P_{21xn} + P_{22xn}) / c_1$	2,439741604835
		kinetic energy $(E_{kin\,21\,xn} + E_{kin\,22\,xn})/c_1^2$	1,419157338706
	After collision	mass $(M_{21k} + M_{22k})$	2,919157338706
		momentum $(P_{21xk} + P_{22xk}) / c_1$	2,439741604835
		kinetic energy $(E_{kin\,21\,xk} + E_{kin\,22\,xk})/c_1^2$	1,419157338706

Tab.2

Ranges $\gamma_V > 1$ and $\gamma_v > 1$. The stationary reference system $O_1x_1y_1z_1$.

Object	Period time	Value	Digit of value
Body 1	Before collision	speed v_{11xn} / c_1	0,965517241379
		mass M_{11n}	3,841143835489
		momentum P_{11xn} / c_1	3,708690599782
		kinetic energy $E_{kin\,11\,xn} / c_1^2$	2,841143835489
	After collision	speed v_{11xk} / c_1	0,903514517939
		mass M_{11k}	2,333409263988
		momentum P_{11xk} / c_1	2,108269146306
		kinetic energy $E_{kin\,11\,xk} / c_1^2$	1,333409263988
Body 2	Before collision	speed v_{12xn} / c_1	0,846153846154
		mass M_{12n}	0,938194187433
		momentum P_{12xn} / c_1	0,793856620136
		kinetic energy $E_{kin\,12\,xn} / c_1^2$	0,438194187433
	After collision	speed v_{12xk} / c_1	0,978882996844
		mass M_{12k}	2,445928758933
		momentum P_{12xk} / c_1	2,394278073612
		kinetic energy $E_{kin\,12\,xk} / c_1^2$	1,945928758933
The system of bodies 1 and 2	Before collision	mass $(M_{11n} + M_{12n})$	4,779338022922
		momentum $(P_{11xn} + P_{12xn}) / c_1$	4,502547219918
		kinetic energy $(E_{kin\,11\,xn} + E_{kin\,12\,xn})/c_1^2$	3,279338022922
	After collision	mass $(M_{11k} + M_{12k})$	4,779338022922
		momentum $(P_{11xk} + P_{12xk}) / c_1$	4,502547219918
		kinetic energy $(E_{kin\,11\,xk} + E_{kin\,12\,xk})/c_1^2$	3,279338022922

As a result of the calculation can draw the following conclusion: in example 1 in the mobile $O_2x_2y_2z_2$ and stationary $O_1x_1y_1z_1$ reference systems before and after the collision the mass, momentum and kinetic energy of the mechanical system of bodies 1 and 2 remain unchanged for the case, when the coefficients of proportionality γ_V and γ_v lie in the ranges $\gamma_V > 1$ and $\gamma_v > 1$, when using the relationships (63) - (65).

Also, by setting the initial data, we can hold the estimated test of choice of dependences (63) - (65) mass, momentum and kinetic energy of the moving body for the case, when the coefficients of proportionality γ_V and γ_v are in the ranges $0 < \gamma_V < 1$ and $0 < \gamma_v < 1$, respectively.

Suppose that: $M_{o1} = 1$, $M_{o2} = 0{,}5$, $V / c_2 = 0{,}5$, $v_{21xn} / c_2 = 0{,}9$, $v_{22xn} / c_2 = 0{,}6$.

For this example 1, with coefficients of proportionality γ_V and γ_v, whose values may be in the ranges $0 < \gamma_V < 1$ and $0 < \gamma_v < 1$, the numerical calculations yield the following results in Tab.3 for the mobile reference system $O_2x_2y_2z_2$ and Tab.4 for the stationary reference system $O_1x_1y_1z_1$.

Tab.3

Ranges $0 < \gamma_V < 1$ and $0 < \gamma_v < 1$. The mobile reference system $O_2x_2y_2z_2$.

Object	Period time	Value	Digit of value
Body 1	Before collision	speed v_{21xn} / c_2	0,9
		mass M_{21n}	0,743294146247
		momentum $P_{21\,xn} / c_2$	0,668964731622
		kinetic energy $E_{kin\,21\,xn} / c_2^2$	0,256705853753
	After collision	speed v_{11xk} / c_2	0,691099932748
		mass M_{21k}	0,822656908881
		momentum $P_{21\,xk} / c_2$	0,568538134403
		kinetic energy $E_{kin\,21\,xk} / c_2^2$	0,177343091119
Body 2	Before collision	speed v_{22xn} / c_2	0,6
		mass M_{22n}	0,428746462856
		momentum $P_{22\,xn} / c_2$	0,257247877714
		kinetic energy $E_{kin\,22\,xn} / c_2^2$	0,071253537144
	After collision	speed v_{22xk} / c_2	1,023729712365
		mass M_{22k}	0,349383700222
		momentum $P_{22\,xk} / c_2$	0,357674474934
		kinetic energy $E_{kin\,22\,xk} / c_2^2$	0,150616299778
The system of bodies 1 and 2	Before collision	mass $(M_{21n} + M_{22n})$	1,172040609103
		momentum $(P_{21\,xn} + P_{22\,xn}) / c_2$	0,926212609336
		kinetic energy $(E_{kin\,21\,xn} + E_{kin\,22\,xn})/c_2^2$	0,327959390897
	After collision	mass $(M_{21k} + M_{22k})$	1,172040609103
		momentum $(P_{21\,xk} + P_{22\,xk}) / c_2$	0,926212609336
		kinetic energy $(E_{kin\,21\,xk} + E_{kin\,22\,xk})/c_2^2$	0,327959390897

Tab.4

Ranges $0 < \gamma_V < 1$ and $0 < \gamma_v < 1$. The stationary reference system $O_1x_1y_1z_1$.

Object	Period time	Value	Digit of value
Body 1	Before collision	speed v_{11xn} / c_2	2,545454545455
		mass M_{11n}	0,365652372423
		momentum P_{11xn} / c_2	0,93075149344
		kinetic energy $E_{kin\,11\,xn} / c_2^2$	0,634347627577
	After collision	speed v_{11xk} / c_2	1,820001331727
		mass M_{11k}	0,481548724902
		momentum P_{11xk} / c_2	0,876419320614
		kinetic energy $E_{kin\,11\,xk} / c_2^2$	0,518451275098
Body 2	Before collision	speed v_{12xn} / c_2	1,571428571429
		mass M_{12n}	0,268437746097
		momentum P_{12xn} / c_2	0,421830743866
		kinetic energy $E_{kin\,12\,xn} / c_2^2$	0,231562253903
	After collision	speed v_{12xk} / c_2	3,121532492927
		mass M_{12k}	0,152541393617
		momentum P_{12xk} / c_2	0,476162916693
		kinetic energy $E_{kin\,12\,xk} / c_2^2$	0,347458606383
The system of bodies 1 and 2	Before collision	mass $(M_{11n} + M_{12n})$	0,63409011852
		momentum $(P_{11xn} + P_{12xn}) / c_2$	1,352582237306
		kinetic energy $(E_{kin\,11\,xn} + E_{kin\,12\,xn})/c_2^2$	0,86590988148
	After collision	mass $(M_{11k} + M_{12k})$	0,63409011852
		momentum $(P_{11xk} + P_{12xk}) / c_2$	1,352582237306
		kinetic energy $(E_{kin\,11\,xk} + E_{kin\,12\,xk})/c_2^2$	0,86590988148

Here also as a result of the calculation can draw the following conclusion:

in example 1 in the mobile $O_2x_2y_2z_2$ and stationary $O_1x_1y_1z_1$ reference systems before and after the collision the mass, momentum and kinetic energy of the mechanical system of bodies 1 and 2 remain unchanged for the case, when the coefficients of proportionality γ_V and γ_v lie in the ranges $0 < \gamma_V < 1$ and $0 < \gamma_V < 1$, when using the relationships (63) - (65).

Substituting formulas (33) and (34) in equations (63) - (65), we can obtain the dependences of mass, momentum and kinetic energy of the moving body on its speed for cases, where the coefficient of proportionality $\gamma_v > 1$ and $0 < \gamma_v < 1$, which are located opposite each other for comparison, and the sign «>» indicates that this is the case when $\gamma_v > 1$, and sign «<» - for the case when $0 < \gamma_v < 1$:

$$M(v)_> = \frac{M_o}{\sqrt{1 - \frac{v^2}{c_1^2}}} \qquad (71)$$

$$P(v)_> = \frac{M_o \cdot v}{\sqrt{1 - \frac{v^2}{c_1^2}}} \qquad (72)$$

$$E_{kin}(v)_> = = M_o \cdot c_1^2 \cdot \left(\frac{1}{\sqrt{1 - \frac{v^2}{c_1^2}}} - 1 \right) \qquad (73)$$

$$M(v)_< = \frac{M_o}{\sqrt{1 + \frac{v^2}{c_2^2}}} \qquad (74)$$

$$P(v)_< = \frac{M_o \cdot v}{\sqrt{1 + \frac{v^2}{c_2^2}}} \qquad (75)$$

$$E_{kin}(v)_< = = M_o \cdot c_2^2 \cdot \left(1 - \frac{1}{\sqrt{1 + \frac{v^2}{c_2^2}}} \right) \qquad (76)$$

Here one can note that a formula similar to formula (74), shows Y.P. Terletsky [4].

The main values of relationships for mass $M(v)_>$ (71), momentum $P(v)_>$ (72) and kinetic energy $E_{kin}(v)_>$ (73) with the coefficient of proportionality

$\gamma_v > 1$ and dependences of mass $M(v)_<$ (74), momentum $P(v)_<$ (75) and kinetic energy $E_{kin}(v)_<$ (76) with a coefficient of proportionality $0 < \gamma_v < 1$ are given in Tab.5 and Tab.6, respectively:

Tab.5

For the coefficient of proportionality $\gamma_v > 1$

Speed v	Mass $M(v)_>$	Momentum $P(v)_>$	Kinetic energy $E_{kin}(v)_>$
$v \ll c_1$	M_o	$M_o \cdot v$	$\frac{M_o \cdot v^2}{2}$
$v < c_1$	has real meaning	has real meaning	has real meaning
$v = c_1$	∞	∞	∞
$v > c_1$	has no real meaning	has no real meaning	has no real meaning

Tab.6

For the coefficient of proportionality $0 < \gamma_v < 1$

Speed v	Mass $M(v)_<$	Momentum $P(v)_<$	Kinetic energy $E_{kin}(v)_<$
$v \ll c_2$	M_o	$M_o \cdot v$	$\frac{M_o \cdot v^2}{2}$
$v < c_2$	has real meaning	has real meaning	has real meaning
$v = c_2$	$\frac{M_o}{\sqrt{2}}$	$\frac{M_o \cdot c_2}{\sqrt{2}}$	$M_o \cdot c_2^2 \cdot \left(1 - \frac{1}{\sqrt{2}}\right)$
$v > c_2$	has real meaning	has real meaning	has real meaning
$v = \infty$	tends to zero	$M_o \cdot c_2$	$M_o \cdot c_2^2$

As seen from tab.5 and tab.6, both the range of possible values of the coefficient of proportionality $\gamma_v > 1$ and $0 < \gamma_v < 1$ are equivalent (both satisfy the boundary condition).

For comparison of formulas (71) - (73) and (74) - (76) give the following graphs:

- graphs of dependence of the mass $M(v)$ of a moving body on the speed v, shown in Fig.10:

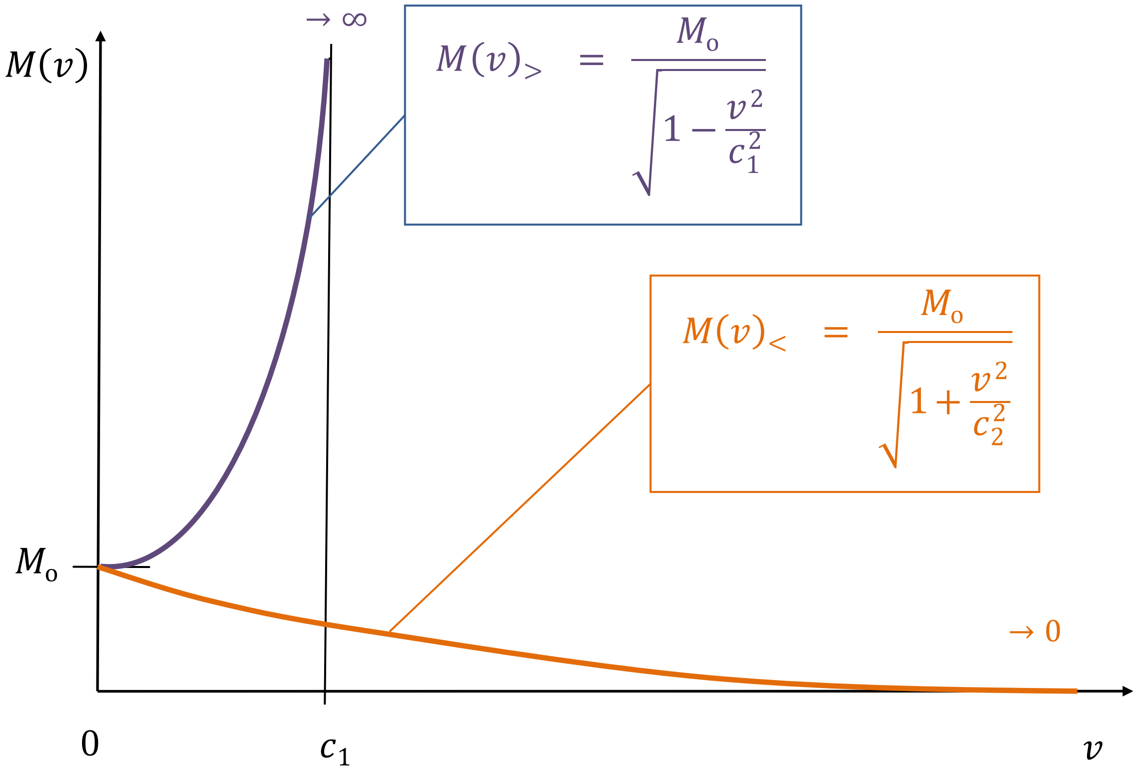

Fig.10

- graphs of dependence of the momentum $P(v)$ of a moving body on the speed v, shown in Fig.11:

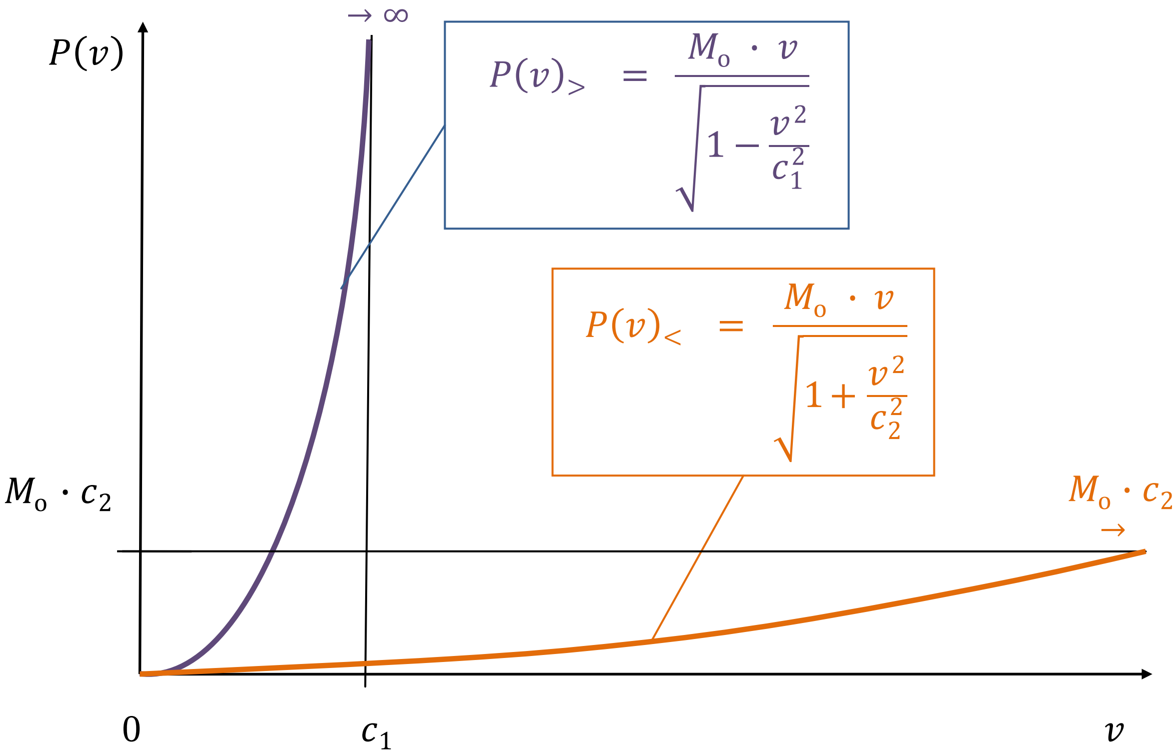

Fig.11

- graphs of dependence of the kinetic energy $E_{kin}(v)$ of a moving body on the speed v, shown in Fig.12:

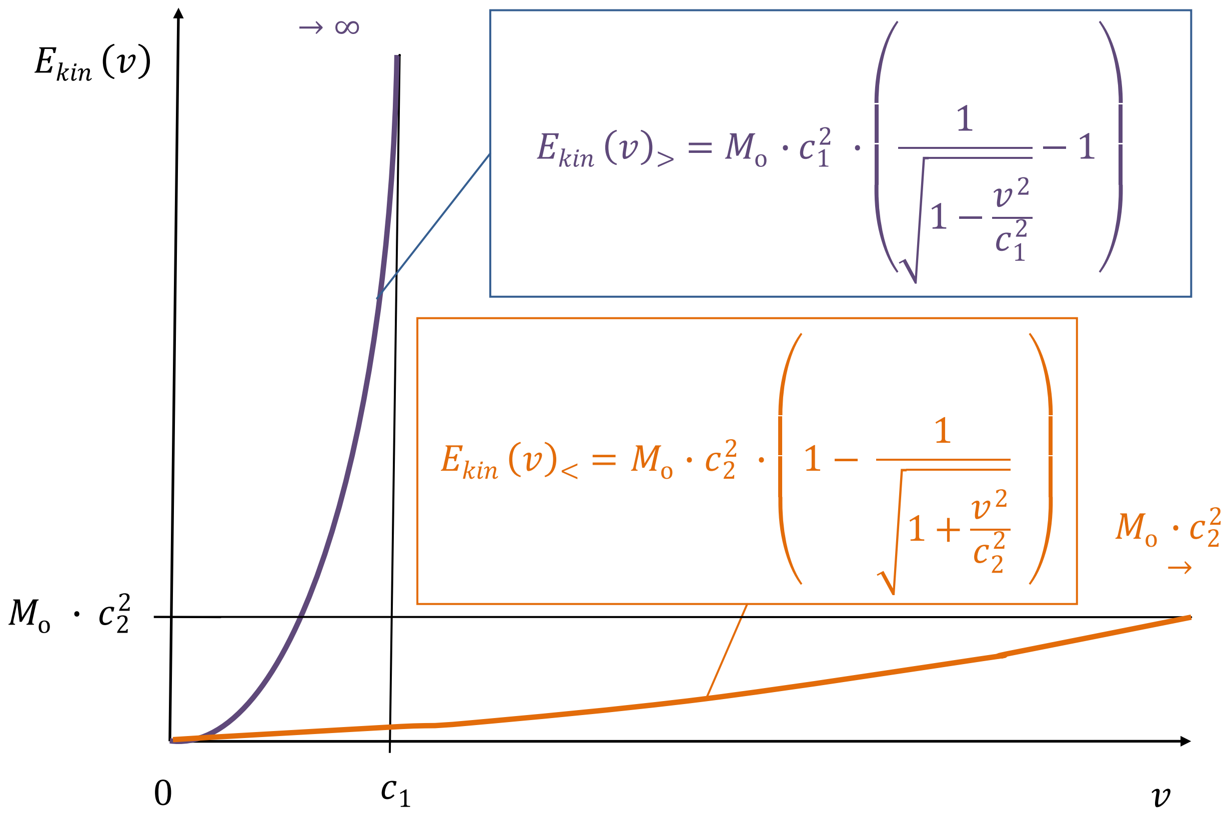

Fig.12

7. Defining values constants c_1 and c_2

The law of conservation of momentum of a closed mechanical system of bodies, connected with the symmetry properties of space - the homogeneity of space [2], states, that the momentum of a closed mechanical system of bodies (which is not acted upon by external forces) is a constant value, ie in any inertial reference system for any point in time the value of the momentum of a closed mechanical system of bodies is a constant value (because there is no external influence).

In the following the above example 2 in the inertial reference system with the help of the special theory of relativity will determine the momentums of the bodies, constituting a closed mechanical system and have been under constant interaction, for two moments of time, then, applying the law of conservation of momentum of a closed mechanical system, will be determined by the values of the constants c_1 and c_2 .

Assume that there are two inertial reference systems, similar to those of reference systems, shown in Fig.1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$, which moves with speed V parallel to the axis O_1x_1 relative to the system $O_1x_1y_1z_1$.

Suppose that there is a closed mechanical system of bodies, shown in Fig.13 and consisting of point bodies 1 and 2, with equal mass M_o at rest, and a string 3.

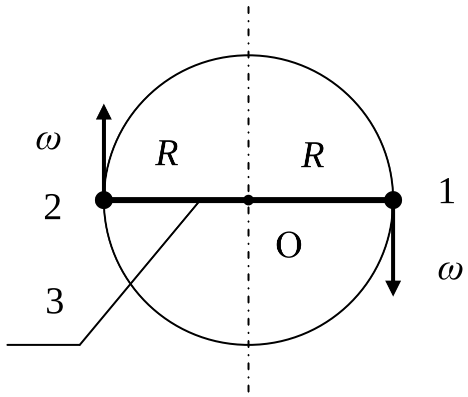

Fig.13

Bodies 1 and 2 are connected by a string 3, the mass of which can be neglected because of its smallness.

Bodies 1 and 2 rotate with angular speed ω around a common center of mass - the point O.

Distance from the point body 1 (body 2) to point O is equal to R.

Let's put a closed mechanical system of bodies 1 and 2 with a string 3 in the moving reference system $O_2x_2y_2z_2$ so, that the point O would be stationary in this reference system, and coincided with the origin O_2, and the rotation of bodies 1 and 2 around it would occur in a clockwise direction in the plane of $O_2x_2y_2$, as shown in Fig.14.

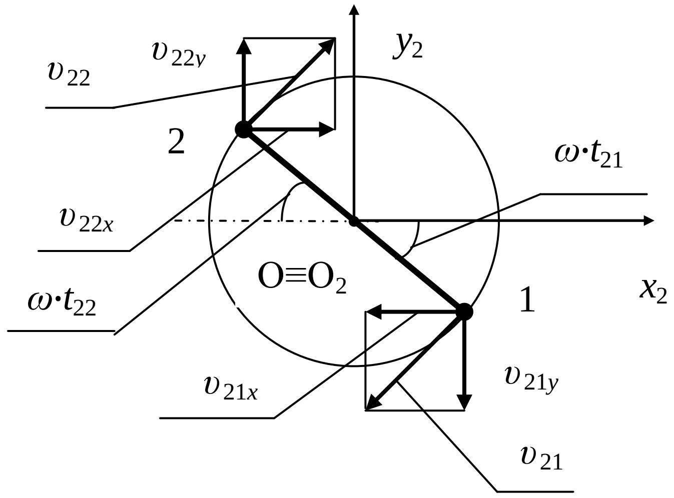

Fig.14

Also assume, that at the start of timing (t_2=0) in the reference system $O_2x_2y_2z_2$ bodies 1 and 2 were on the axis O_2x_2 , with the body 1 had a positive coordinate, and the body 2 - negative.

In the mobile reference system $O_2x_2y_2z_2$ at any time t_2 bodies 1 and 2 will have the speeds υ_{21} and υ_{22}, equal v_R :

$$v_{21} = v_{22} = v_R = \omega \cdot R \qquad (77)$$

In this case, the projections υ_{21x} and υ_{21y} of speed of the body 1 and the projections υ_{22x} and υ_{22y} of speed of the body 2 on the axis O_2x_2 and O_2y_2 , respectively, for times t_{21} and t_{22} will be equal to:

$$v_{21\,x} = -\left[v_R \cdot \sin(\omega \cdot t_{21})\right] \qquad (78)$$

$$v_{21\,y} = -\left[v_R \cdot \cos(\omega \cdot t_{21})\right] \qquad (79)$$

$$v_{22\,x} = v_R \cdot \sin(\omega \cdot t_{22}) \qquad (80)$$

$$v_{22\,y} = v_R \cdot \cos(\omega \cdot t_{22}) \qquad (81)$$

The relationship between the coordinates x_{21} and y_{21} of the body 1 depending on time t_{21} and the relationship between the coordinates x_{22} and y_{22} of the body 2 depending on the time t_{22} in the mobile reference system $O_2x_2y_2z_2$ can be written as:

$$x_{21} = R \cdot \cos(\omega \cdot t_{21}) \qquad (82)$$

$$y_{21} = -\left[R \cdot \sin(\omega \cdot t_{21})\right] \qquad (83)$$

$$x_{22} = -[R \cdot \cos(\omega \cdot t_{22})] \tag{84}$$

$$y_{22} = R \cdot \sin(\omega \cdot t_{22}) \tag{85}$$

Based on the equations (1) and (3), we can write the relationships between:

- coordinates x_{11} and y_{11} of the body 1 at time t_{11} in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{21} and y_{21} of the body 1 in the mobile reference system $O_2x_2y_2z_2$ at time t_{21} , which corresponds to the time t_{11} in the stationary reference system $O_1x_1y_1z_1$:

$$x_{11} = \gamma_V \cdot [x_{21} + (V \cdot t_{21})] \tag{86}$$

$$y_{11} = y_{21} \tag{87}$$

- coordinates x_{12} and y_{12} of the body 2 at time t_{12} in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{22} and y_{22} of the body 2 in the mobile reference system $O_2x_2y_2z_2$ at time t_{22} , which corresponds to the time t_{12} in the stationary reference system $O_1x_1y_1z_1$:

$$x_{12} = \gamma_V \cdot [x_{22} + (V \cdot t_{22})] \tag{88}$$

$$y_{12} = y_{22} \tag{89}$$

Using formula (5) relationship between the values of the times t_{11} and t_{21} , t_{12} and t_{22} will look like this:

$$t_{11} = \frac{(\gamma_V^2 - 1) \cdot x_{21}}{\gamma_V \cdot V} + (\gamma_V \cdot t_{21}) \tag{90}$$

$$t_{12} = \frac{(\gamma_V^2 - 1) \cdot x_{22}}{\gamma_V \cdot V} + (\gamma_V \cdot t_{22}) \tag{91}$$

In the considered example 2, we are interested in the position of bodies 1 and 2 in the stationary reference system $O_1x_1y_1z_1$ at the same time, ie where:

$$t_{11} = t_{12} \tag{92}$$

Then equation (92) taking into account formulas (82), (84), (86), (88), (90) and (91) becomes:

$$\frac{(\gamma_V^2 - 1) \cdot R \cdot \cos(\omega \cdot t_{21})}{\gamma_V \cdot V} + (\gamma_V \cdot t_{21}) =$$

$$= \frac{(1 - \gamma_V^2) \cdot R \cdot \cos(\omega \cdot t_{22})}{\gamma_V \cdot V} + (\gamma_V \cdot t_{22}) \tag{93}$$

In the mobile reference system $O_2x_2y_2z_2$ when performing the condition (92) it is interesting position of the bodies 1 and 2 at the time t_{2p} , when:

$$t_{21} = t_{22} = t_{2p} \qquad (94)$$

Substituting condition (94) in equation (93) for the case when $(\omega \cdot t_{2p}) < \pi$, we obtain:

$$\omega \cdot t_{2p} = \frac{\pi}{2} \qquad (95)$$

Ie to meet the conditions (92) and (94) during the time t_{2p} the bodies 1 and 2 should be on a line parallel to the axis O_2y_2.

Also in the mobile reference system $O_2x_2y_2z_2$ when performing the condition (92) it is interesting position of body 2 when finding the body 1 on the axis O_2x_2 at time t_{21} , equal to t_{21h} , where:

$$t_{21h} = 0 \qquad (96)$$

The value of time t_{22} , when performing the conditions (92) and (96), denote t_{22h} , for which the equation (93) becomes:

$$t_{22h} = \left(1 - \frac{1}{\gamma_V^2}\right) \cdot [1 + \cos(\omega \cdot t_{22h})] \cdot \frac{R}{V} \qquad (97)$$

or:

$$\omega \cdot t_{22h} = \left(1 - \frac{1}{\gamma_V^2}\right) \cdot [1 + \cos(\omega \cdot t_{22h})] \cdot \frac{v_R}{V} \qquad (98)$$

As seen from equation (98), depending on the value of the coefficient of proportionality γ_V the value of time t_{22h} can be:

- $t_{22h} > 0$ when $\gamma_V > 1$;
- $t_{22h} < 0$ when $0 < \gamma_V < 1$;
- $t_{22h} = 0$ when $\gamma_V = 1$.

Now we can begin to use the law of conservation of momentum for the preparation of equations.

Consider two points in time in the stationary reference system $O_1x_1y_1z_1$.

As a first point in time we choose t_{1p} .

Under the terms of (92), (94) and (95) in the moving mobile reference system $O_2x_2y_2z_2$ at time t_{2p} the bodies 1 and 2 are on a line parallel to the axis O_2y_2 and in the stationary reference system $O_1x_1y_1z_1$ the bodies 1 and 2 will be on a line parallel to the axis O_1y_1 at time t_{11} (t_{12}), equal t_{1p} and which corresponds to the time t_{2p} in the mobile reference system $O_2x_2y_2z_2$.

As shown in Fig.15, according to equations (95), (78) - (81) in the mobile reference system $O_2x_2y_2z_2$ at time t_{2p} the bodies 1 and 2, respectively, have the following values of the projections $v_{21\,xp}$, $v_{21\,yp}$ and $v_{22\,xp}$, $v_{22\,yp}$ of speeds of his movement on the axis O_2x_2 and O_2y_2 :

$$v_{21\,xp} = -\,v_R \qquad (99)$$

$$v_{21\,yp} = 0 \qquad (100)$$

$$v_{22\,xp} = v_R \qquad (101)$$

$$v_{22\,yp} = 0 \qquad (102)$$

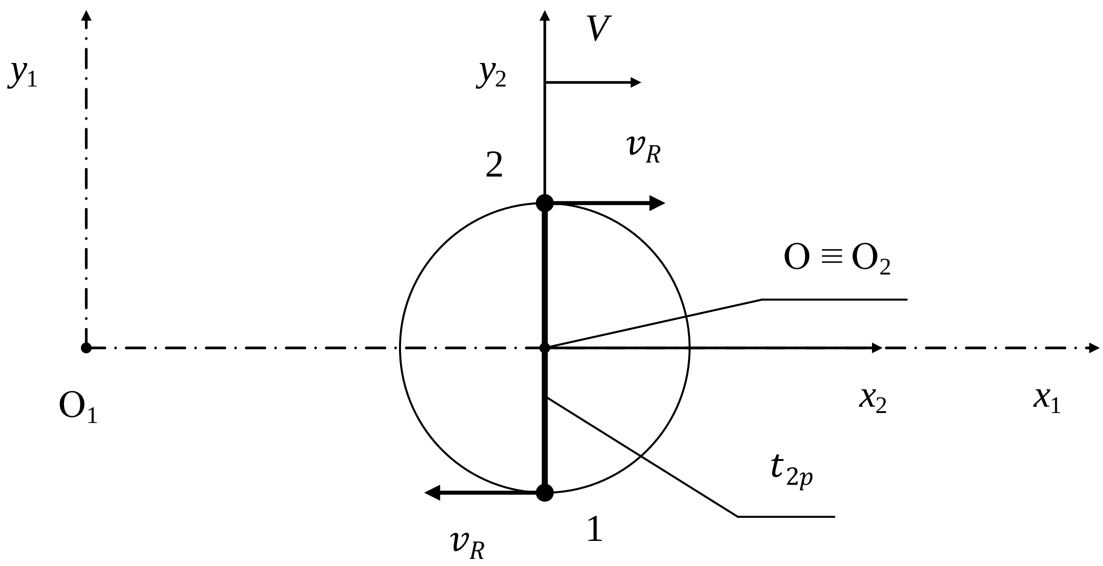

Fig.15

Then, on the basis of formulas (7), (9) and equalities (99) - (102), in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} the body 1 and the body 2, respectively, will have the following values of the projections $v_{11\,xp}$, $v_{11\,yp}$ and $v_{12\,xp}$, $v_{12\,yp}$ of speeds of his movement on the axis O_1x_1 and O_1y_1 :

$$v_{11\,xp} = \frac{V - v_R}{1 - \frac{(\gamma_V^2 - 1)\cdot v_R}{\gamma_V^2 \cdot V}} \quad (103)$$

$$v_{11\,yp} = 0 \quad (104)$$

$$v_{12\,xp} = \frac{V + v_R}{\frac{(\gamma_V^2 - 1)\cdot v_R}{\gamma_V^2 \cdot V} + 1} \quad (105)$$

$$v_{12\,yp} = 0 \quad (106)$$

Hence, using formula (64), may be noted that in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} the body 1 and the body 2, respectively, will have the following values of the projections $P_{11\,xp}$, $P_{11\,yp}$ and $P_{12\,xp}$, $P_{12\,yp}$ of momentums on the axis O_1x_1 and O_1y_1:

$$P_{11\,xp} = M_{\text{o}} \cdot \gamma_{v11\,x\text{p}} \cdot v_{11\,x\text{p}} \quad (107)$$

$$P_{12\,xp} = M_{\text{o}} \cdot \gamma_{v12\,x\text{p}} \cdot v_{12\,x\text{p}} \quad (108)$$

$$P_{11\,yp} = 0 \quad (109)$$

$$P_{12\,yp} = 0 \quad (110)$$

where: $\gamma_{v11\,xp}$ and $\gamma_{v12\,xp}$ - the coefficients of proportionality of the speed V, equal to $v_{11\,xp}$ and $v_{12\,xp}$ respectively.

As a second point in time we choose t_{1h} .

Under the terms of (92) and (96) in the mobile reference system $O_2x_2y_2z_2$ at time $t_{21h} = 0$ the body 1 will be located on the axis O_2x_2 , and in the stationary reference system $O_1x_1y_1z_1$ the body 1 will be located on the axis O_1x_1 at time t_{11} (t_{12}), equal t_{1h} and which corresponds to the time $t_{21h} = 0$ in the mobile reference system $O_2x_2y_2z_2$.

Moreover in the mobile reference system $O_2x_2y_2z_2$ according to equation (98), when the value of the coefficient of proportionality $\gamma_V \neq 1$, the body 2 can not be on the axis O_2x_2 at time t_{22h} , which corresponds to the time t_{1h} in the stationary reference system $O_1x_1y_1z_1$.

Ie the body 1 is located on the axis O_1x_1 in a the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} , which corresponds to the time $t_{21h} = 0$ in the mobile reference system $O_2x_2y_2z_2$, and at the time t_{1h} the body 2 can not lie on the axis O_2x_2 (with a coefficient of proportionality $\gamma_V \neq 1$) .

As shown in Fig.16, in the mobile reference system $O_2x_2y_2z_2$ the body 1 at the time $t_{21\,h} = 0$ and the body 2 at the time t_{22h} respectively have projections $v_{21\,xh}$, $v_{21\,yh}$ and $v_{22\,xh}$, $v_{22\,yh}$ of speeds of his movement on the axis O_2x_2 and O_2y_2 , so that:

$$v_{21\,xh} = 0 \qquad (111)$$

$$v_{21\,yh} = -\,v_R \qquad (112)$$

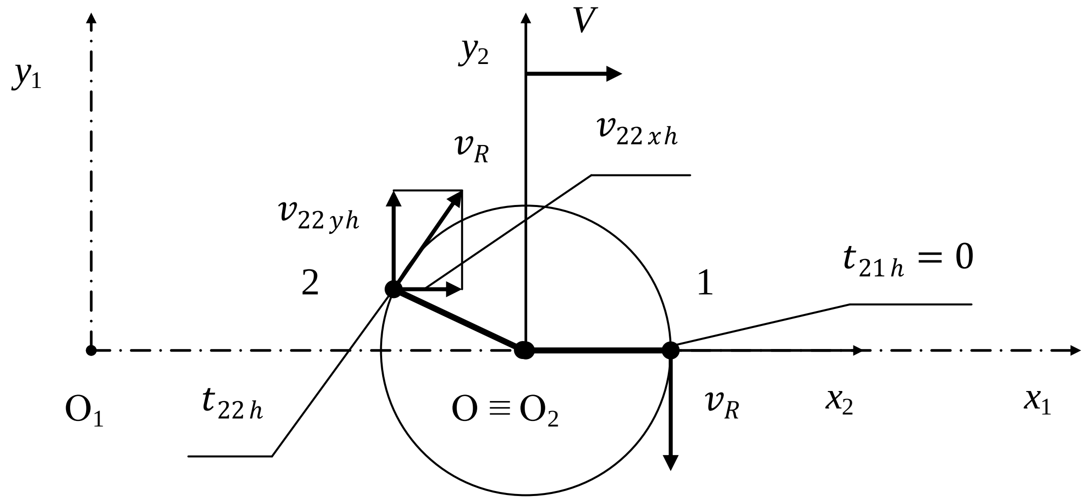

Fig.16

Then, on the basis of formulas (7), (9) and equations (111), (112), in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} the body 1 and the body 2, respectively, will have the values of the projections $v_{11\,xh}$, v_{11yh} and $v_{12\,xh}$, $v_{12\,yh}$ of speeds of his movement on the axis O_1x_1 and O_1y_1 :

$$v_{11\,xh} = V \qquad (113)$$

$$v_{11\,yh} = -\,\frac{v_R}{\gamma_V} \qquad (114)$$

$$v_{12\,xh} = \frac{V + v_{22\,xh}}{\dfrac{(\gamma_V^2 - 1)\cdot v_{22\,xh}}{\gamma_V^2 \cdot V} + 1} \qquad (115)$$

$$v_{12yh} = \frac{v_{22yh}}{\frac{(\gamma_V^2 - 1) \cdot v_{22xh}}{\gamma_V \cdot V} + \gamma_V} \qquad (116)$$

Given equation (98), we note that:

- with the coefficient of proportionality $\gamma_V > 1$ the time $t_{22h} > 0$, so the projection v_{22yh} of the speed will be the direction of the axis O_2y_2;

- with the coefficient of proportionality $0 < \gamma_V < 1$ the time $t_{22h} < 0$, so the projection v_{22yh} of the speed will be the direction opposite to the axis O_2y_2 .

From equations (80) and (81) it follows that:

$$v_{22xh}{}^2 + v_{22yh}{}^2 = v_R{}^2 \qquad (117)$$

Using formula (64), may be noted that in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} the body 1 and the body 2, respectively, will have the following values of the projections P_{11xh} , P_{11yh} and P_{12xh} , P_{12yh} of momentums on the axis O_1x_1 and O_1y_1 :

$$P_{11xh} = M_\text{o} \cdot \gamma_{v11h} \cdot v_{11\text{x}h} \qquad (118)$$

$$P_{12xh} = M_\text{o} \cdot \gamma_{v12h} \cdot v_{12\text{x}h} \qquad (119)$$

$$P_{11yh} = M_\text{o} \cdot \gamma_{v11h} \cdot v_{11\text{y}h} \qquad (120)$$

$$P_{12yh} = M_\text{o} \cdot \gamma_{v12h} \cdot v_{12\text{y}h} \qquad (121)$$

where: γ_{v11h} and γ_{v12h} - the coefficients of proportionality in the speed V, equal to v_{11h} and v_{12h} respectively, so that:

$$v_{11h}^2 = v_{11xh}^2 + v_{11yh}^2 \qquad (122)$$

$$v_{12h}^2 = v_{12xh}^2 + v_{12yh}^2 \qquad (123)$$

Due to the fact, that the mechanical system of the bodies 1 and 2 (and string 3) is closed, the law of conservation of momentum can write the following equations for the moments of times t_{1p} and t_{1h} :

$$P_{11xp} + P_{12xp} = P_{11xh} + P_{12xh}$$

$$P_{11yp} + P_{12yp} = P_{11yh} + P_{12yh}$$

or:

$$\left(M_\text{o} \cdot \gamma_{v11xp} \cdot v_{11x\text{p}}\right) + \left(M_\text{o} \cdot \gamma_{v12xp} \cdot v_{12x\text{p}}\right) =$$

$$= (M_{\text{o}} \cdot \gamma_{v11\,h} \cdot v_{11\,xh}) + (M_{\text{o}} \cdot \gamma_{v12\,h} \cdot v_{12\,xh}) \qquad (124)$$

$$0 = (M_{\text{o}} \cdot \gamma_{v11\,h} \cdot v_{11\,yh}) + (M_{\text{o}} \cdot \gamma_{v12\,h} \cdot v_{12\,yh}) \qquad (125)$$

Having obtained equation (124) and (125), can determine the conditions of implementation of the law of conservation of momentum for example 2 in the stationary reference system $O_1x_1y_1z_1$ for the case when the values of the coefficient of proportionality γ_V (also γ_v) lie in the range $\gamma_V \geq 1$ (and $\gamma_v \geq 1$).

Equations (124) and (125) taking into account formula (33) take the form:

$$\frac{M_{\text{o}} \cdot v_{11\,x\text{p}}}{\sqrt{1 - \frac{v_{11\,x\text{p}}^2}{c_1^2}}} + \frac{M_{\text{o}} \cdot v_{12\,x\text{p}}}{\sqrt{1 - \frac{v_{12\,x\text{p}}^2}{c_1^2}}} = \frac{M_{\text{o}} \cdot v_{11\,xh}}{\sqrt{1 - \frac{v_{11\,xh}^2 + v_{11\,yh}^2}{c_1^2}}} + \frac{M_{\text{o}} \cdot v_{12xh}}{\sqrt{1 - \frac{v_{12\,xh}^2 + v_{12\,yh}^2}{c_1^2}}} \qquad (126)$$

$$0 = \frac{M_{\text{o}} \cdot v_{11\,yh}}{\sqrt{1 - \frac{v_{11\,xh}^2 + v_{11\,yh}^2}{c_1^2}}} + \frac{M_{\text{o}} \cdot v_{12\,yh}}{\sqrt{1 - \frac{v_{12\,xh}^2 + v_{12\,yh}^2}{c_1^2}}} \qquad (127)$$

Formulas (103) - (106) and (113) - (116) using the formula (33) can be written:

$$v_{11\,xp} = \frac{V - v_R}{1 - \frac{V \cdot v_R}{c_1^2}} \qquad (128)$$

$$v_{12\,xp} = \frac{V + v_R}{1 + \frac{V \cdot v_R}{c_1^2}} \qquad (129)$$

$$v_{11\,xh} = V \qquad (113)$$

$$v_{11\,yh} = -\left(v_R \cdot \sqrt{1 - \frac{V^2}{c_1^2}}\right) \qquad (130)$$

$$v_{12\,xh} = \frac{V + v_{22\,xh}}{1 + \frac{V \cdot v_{22\,xh}}{c_1^2}} \qquad (131)$$

$$v_{12\,yh} = \frac{v_{22\,yh} \cdot \sqrt{1 - \frac{V^2}{c_1^2}}}{1 + \frac{V \cdot v_{22\,xh}}{c_1^2}} \qquad (132)$$

By inserting the projections v_{11xp} , v_{12xp} , v_{11xh} , v_{11yh} , v_{12xh} and v_{12yh} of speeds of formulas (113), (128) - (132) in equations (126) and (127) and using the formula (117), we obtain:

$$\frac{M_o \cdot (V - v_R)}{\sqrt{1-\frac{{v_R}^2}{c_1^2}} \cdot \sqrt{1-\frac{V^2}{c_1^2}}} + \frac{M_o \cdot (V + v_R)}{\sqrt{1-\frac{{v_R}^2}{c_1^2}} \cdot \sqrt{1-\frac{V^2}{c_1^2}}} = \frac{M_o \cdot V}{\sqrt{1-\frac{{v_R}^2}{c_1^2}} \cdot \sqrt{1-\frac{V^2}{c_1^2}}} + \frac{M_o \cdot (V + v_{22xh})}{\sqrt{1-\frac{{v_R}^2}{c_1^2}} \cdot \sqrt{1-\frac{V^2}{c_1^2}}} \qquad (133)$$

$$0 = -\frac{M_o \cdot v_R}{\sqrt{1-\frac{{v_R}^2}{c_1^2}}} + \frac{M_o \cdot v_{22yh}}{\sqrt{1-\frac{{v_R}^2}{c_1^2}}} \qquad (134)$$

or:

$$V - v_R + V + v_R = V + V + v_{22xh} \qquad (135)$$

$$0 = -v_R + v_{22yh} \qquad (136)$$

From equations (135) and (136) obtain the necessary conditions (the values of the projections v_{22xh} and v_{22yh} of speeds), which in the example 2 will be implemented by law of conservation of momentum in the stationary inertial reference system $O_1x_1y_1z_1$ for the case when the values of the coefficient of proportionality γ_V are in the range $\gamma_V \geq 1$:

$$v_{22xh} = 0 \qquad (137)$$

$$v_{22yh} = v_R \qquad (138)$$

From (137) and (138) it follows, that the values of projections v_{22xh} and v_{22yh} of speeds do not depend on the magnitude of the speed V (and, consequently, do not depend on the magnitude of the coefficient of proportionality γ_V).

Substituting conditions (137) and (138) in equations (80) and (81), we obtain:

$$t_{22h} = t_{21h} = 0 \qquad (139)$$

And substituting equation (139) in the formula (98):

$$\omega \cdot 0 \ = \left(1 - \frac{1}{\gamma_V^2}\right) \cdot \ [1 + \ 1] \ \cdot \frac{v_R}{V} \qquad (140)$$

will have another condition for the implementation of the law of conservation of momentum in the stationary inertial reference system $O_1x_1y_1z_1$ for example 2:

$$\gamma_V \ = 1 \qquad (141)$$

Thus, we can conclude, that in a closed mechanical system of bodies, considered in example 2, for the values of the coefficient of proportionality $\gamma_V > 1$ the law of conservation of momentum is not satisfied.

Similarly, using equation (124) and (125), can determine the conditions of implementation of the law of conservation of momentum for example 2 in the stationary reference system $O_1x_1y_1z_1$ for the case, when the values of the coefficient of proportionality γ_V (also γ_v) lie in the range $0 < \gamma_V \ \leq 1$ (and $0 \ < \ \gamma_v \ \leq \ 1$).

Equations (124) and (125) taking into account formula (34) take the form:

$$\frac{M_\text{o} \cdot v_{11\,x\text{p}}}{\sqrt{1 + \frac{v_{11\,x\text{p}}^2}{c_2^2}}} + \frac{M_\text{o} \cdot v_{12\,x\text{p}}}{\sqrt{1 + \frac{v_{12\,x\text{p}}^2}{c_2^2}}} = \frac{M_\text{o} \ \cdot \ v_{11\,\text{x}h}}{\sqrt{1 + \frac{v_{11\,\text{x}h}^2 + v_{11\,yh}^2}{c_2^2}}} + \frac{M_\text{o} \ \cdot \ v_{12\,\text{x}h}}{\sqrt{1 + \frac{v_{12\,\text{x}h}^2 + v_{12\,yh}^2}{c_2^2}}} \qquad (142)$$

$$0 = \frac{M_\text{o} \ \cdot \ v_{11\,\text{y}h}}{\sqrt{1 + \frac{v_{11\,\text{x}h}^2 + v_{11\,yh}^2}{c_2^2}}} + \frac{M_\text{o} \ \cdot \ v_{12\,\text{x}h}}{\sqrt{1 + \frac{v_{12\,\text{x}h}^2 + v_{12\,yh}^2}{c_2^2}}} \qquad (143)$$

Formulas (103) - (106) and (113) - (116) using the formula (34) can be written:

$$v_{11\,xp} \ = \frac{V \ - \ v_R}{1 + \frac{V \ \cdot v_R}{c_2^2}} \qquad (144)$$

$$v_{12\,xp} \ = \frac{V + v_R}{1 \ - \ \frac{V \ \cdot v_R}{c_2^2}} \qquad (145)$$

$$v_{11\,xh} = V \qquad (113)$$

$$v_{11\,yh} = -\left(v_R \cdot \sqrt{1+\frac{V^2}{c_2^2}}\right) \qquad (146)$$

$$v_{12\,xh} = \frac{V + v_{22\,xh}}{1 - \frac{V \cdot v_{22\,xh}}{c_2^2}} \qquad (147)$$

$$v_{12\,yh} = \frac{v_{22\,yh} \cdot \sqrt{1+\frac{V^2}{c_2^2}}}{1 - \frac{V \cdot v_{22\,xh}}{c_2^2}} \qquad (148)$$

By inserting the projections $v_{11\,xp}$, $v_{12\,xp}$, $v_{11\,xh}$, $v_{11\,yh}$, $v_{12\,xh}$ and $v_{12\,yh}$ of speeds of formulas (113), (144)-(148) in equations (142) and (143) and using the formula (117), we obtain:

$$\frac{M_o \cdot (V - v_R)}{\sqrt{1+\frac{{v_R}^2}{c_2^2}} \cdot \sqrt{1+\frac{V^2}{c_2^2}}} + \frac{M_o \cdot (V + v_R)}{\sqrt{1+\frac{{v_R}^2}{c_2^2}} \cdot \sqrt{1+\frac{V^2}{c_2^2}}} =$$

$$= \frac{M_o \cdot V}{\sqrt{1+\frac{{v_R}^2}{c_2^2}} \cdot \sqrt{1+\frac{V^2}{c_2^2}}} + \frac{M_o \cdot (V + v_{22\,xh})}{\sqrt{1+\frac{{v_R}^2}{c_2^2}} \cdot \sqrt{1+\frac{V^2}{c_2^2}}} \qquad (149)$$

$$0 = -\frac{M_o \cdot v_R}{\sqrt{1+\frac{{v_R}^2}{c_2^2}}} + \frac{M_o \cdot v_{22\,yh}}{\sqrt{1+\frac{{v_R}^2}{c_2^2}}} \qquad (150)$$

or:

$$V - v_R + V + v_R = V + V + v_{22\,xh} \qquad (135)$$

$$0 = -v_R + v_{22\,yh} \qquad (136)$$

From equations (135) and (136) obtain the necessary conditions (the values of the projections $v_{22\,xh}$ and $v_{22\,yh}$ of speeds), which in the example 2 will be implemented by law of conservation of momentum in the stationary inertial reference system $O_1x_1y_1z_1$ for the case when the values of the coefficient of proportionality γ_V are in the range $0 < \gamma_V \leq 1$:

$$v_{22xh} = 0 \quad (137)$$

$$v_{22yh} = v_R \quad (138)$$

Equalities (137) and (138), as has been shown in considering the case, when the value of the coefficient of proportionality γ_V are in the range $\gamma_V \geq 1$, lead to the same condition for the implementation of the law of conservation of momentum for example 2 in the stationary reference system $O_1x_1y_1z_1$ for the case, when the values of the coefficient of proportionality γ_V are in the range $0 < \gamma_V \leq 1$:

$$\gamma_V = 1 \quad (141)$$

Consequently, we can make a similar conclusion, that in a closed mechanical system of bodies, considered in example 2, for the values of the coefficient of proportionality $0 < \gamma_V < 1$ the law of conservation of momentum also is not satisfied.

Summarizing the findings, we note, that in a closed mechanical system of bodies, considered in example 2, for the values of the coefficient of proportionality $\gamma_V > 1$ and $0 < \gamma_V < 1$ the law of conservation of momentum is not satisfied.

The law of conservation of momentum will be carried out only, if the coefficient of proportionality γ_V equal to 1.

In the case of the obligation to fulfill the law of conservation of momentum of a closed mechanical system of bodies, considered in example 2, based on the formulas (26) - (28), and given, that the coefficient of proportionality $\gamma_V = 1$, constants c_1 and c_2 will have the following meanings:

$$c_1 = \pm\infty \quad (151)$$

$$c_2 = \pm\infty \quad (152)$$

8. Score quantities of momentums in example 3

Given the possible observation, that in example 2 the failure of the law of conservation of momentum can be attributed to the assumptions of infinitesimal mass of the string 3, consider the example 3.

Example 3 differs from example 2 in that in example 3 the mass of the string 3 is not infinitely small.

For example 3 will try to assess the impact of magnitude of the momentum of the string 3 on the magnitude of the momentum of bodies 1 and 2 and string 3.

Assume that there are two inertial reference systems, similar to those of reference systems, shown in Fig.1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$, which moves with speed V parallel to the axis O_1x_1 relative to the system $O_1x_1y_1z_1$.

Suppose that there is a closed mechanical system of bodies, shown in Fig.13 and consisting of point bodies 1 and 2, with equal mass M_o at rest, and a string 3.

Bodies 1 and 2 are connected by a string 3, which has a mass of uniformly distributed along its length and equal to m_o at rest.

Bodies 1 and 2 rotate with angular speed ω around a common center of mass - the point O.

Distance from the point body 1 (body 2) to point O is equal to R.

Let's put a closed mechanical system of bodies 1 and 2 with a string 3 in the moving reference system $O_2x_2y_2z_2$ so, that the point O would be stationary in this reference system, and coincided with the origin O_2, and the rotation of bodies 1 and 2 around it would occur in a clockwise direction in the plane of $O_2x_2y_2$, as shown in Fig.14.

Also assume, that at the start of timing (t_2=0) in the reference system $O_2x_2y_2z_2$ bodies 1 and 2 were on the axis O_2x_2, with the body 1 had a positive coordinate, and the body 2 - negative.

In the mobile reference system $O_2x_2y_2z_2$ at any time t_2 bodies 1 and 2 will have the speeds υ_{21} and υ_{22}, equal v_R:

$$v_{21} = v_{22} = v_R = \omega \cdot R \qquad (77)$$

In this case, the projections υ_{21x} and υ_{21y} of speed of the body 1 and the projections υ_{22x} and υ_{22y} of speed of body 2 on the axis O_2x_2 and O_2y_2 , respectively, for time t_2 will be equal to:

$$v_{21x} = -[v_R \cdot \sin(\omega \cdot t_2)] \quad (153)$$

$$v_{21y} = -[v_R \cdot \cos(\omega \cdot t_2)] \quad (154)$$

$$v_{22x} = v_R \cdot \sin(\omega \cdot t_2) \quad (155)$$

$$v_{22y} = v_R \cdot \cos(\omega \cdot t_2) \quad (156)$$

The relationship between the coordinates x_{21} and y_{21} of the body 1 and the relationship between the coordinates x_{22} and y_{22} body 2 depending on the time t_2 in the mobile reference system $O_2x_2y_2z_2$ can be written as:

$$x_{21} = R \cdot \cos(\omega \cdot t_2) \quad (157)$$

$$y_{21} = -[R \cdot \sin(\omega \cdot t_2)] \quad (158)$$

$$x_{22} = -[R \cdot \cos(\omega \cdot t_2)] \quad (159)$$

$$y_{22} = R \cdot \sin(\omega \cdot t_2) \quad (160)$$

Similarly, for the mobile reference system $O_2x_2y_2z_2$ you can get the dependencies:

- dependencies of the projections $v_{21x\rho i}$ and $v_{21y\rho i}$ of the speed of the *i*-point of the string 3, which is located at a distance ρ_i from point O on the segment from point O to the body 1, on the axis O_2x_2 and O_2y_2 on the time t_2 :

$$v_{21x\rho i} = -\left[v_R \cdot \frac{\rho_i}{R} \cdot \sin(\omega \cdot t_2)\right] \quad (161)$$

$$v_{21y\rho i} = -\left[v_R \cdot \frac{\rho_i}{R} \cdot \cos(\omega \cdot t_2)\right] \quad (162)$$

- dependencies of the projections $v_{22x\rho j}$ and $v_{22y\rho j}$ of the speed of the *j*-point of the string 3, which is located at a distance ρ_j from point O on the segment from point O to the body 2, on the axis O_2x_2 and O_2y_2 on the time t_2 :

$$v_{22x\rho j} = v_R \cdot \frac{\rho_j}{R} \cdot \sin(\omega \cdot t_2) \quad (163)$$

$$v_{22y\rho j} = v_R \cdot \frac{\rho_j}{R} \cdot \cos(\omega \cdot t_2) \quad (164)$$

- dependencies of the values of the coordinates $x_{21\rho i}$ and $y_{21\rho i}$ of *i*-point of the string 3 and the coordinates $x_{22\rho j}$ and $y_{22\rho j}$ of *j*-point of the string 3 on the time t_2 :

$$x_{21\,\rho i} = \rho_i \cdot \cos(\omega \cdot t_2) \quad (165)$$

$$y_{21\,\rho i} = -[\rho_i \cdot \sin(\omega \cdot t_2)] \quad (166)$$

$$x_{22\,\rho j} = -[\rho_j \cdot \cos(\omega \cdot t_2)] \quad (167)$$

$$y_{22\,\rho j} = \rho_j \cdot \sin(\omega \cdot t_2) \quad (168)$$

Now you can proceed to consider the movement of bodies 1 and 2 and string 3 in the stationary reference system $O_1x_1y_1z_1$.

Suppose that, as shown in Fig.17, the mobile inertial reference system $O_2x_2y_2z_2$ moves with speed V relative to the stationary reference system $O_1x_1y_1z_1$, where in the two systems as the origin of time (t_1=0 and t_2=0) is selected such a time, when origins O_1 and O_2 of these systems are the same (ie, when points O_1, O_2 and O are the same).

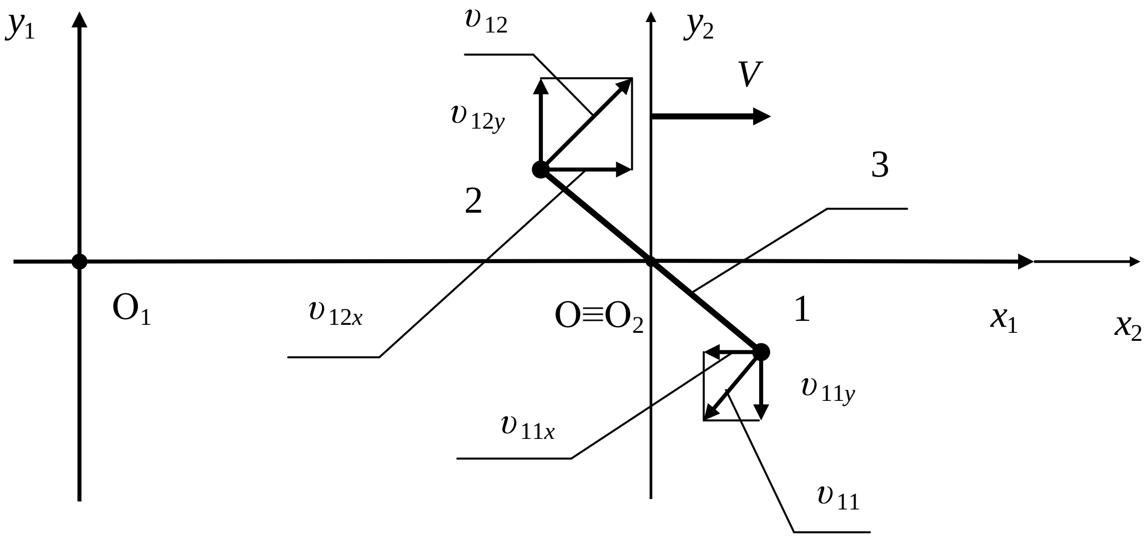

Fig.17

From equations (1) - (3), (5) - (7), (9), to consider the motion of the body 1, we can write the following:

- relationships between coordinates x_{11} and y_{11} of the body 1 at time t_1 in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{21} and y_{21} of the body 1 in the mobile reference system $O_2x_2y_2z_2$ at time t_2, which corresponds to the time t_1 in the stationary reference system $O_1x_1y_1z_1$:

$$x_{11} = \gamma_V \cdot [x_{21} + (V \cdot t_2)] \quad (169)$$

$$x_{21} = \gamma_V \cdot [x_{11} - (V \cdot t_1)] \quad (170)$$

$$y_{11} = y_{21} \tag{87}$$

- relationship between the values of times t_1 and t_2 in describing the motion of the body 1:

$$t_1 = \frac{(\gamma_V^2 - 1) \cdot x_{21}}{\gamma_V \cdot V} + (\gamma_V \cdot t_2) \tag{171}$$

$$t_2 = \frac{(1 - \gamma_V^2) \cdot x_{11}}{\gamma_V \cdot V} + (\gamma_V \cdot t_1) \tag{172}$$

while taking into account the equation (157) formula (171) becomes:

$$t_1 = \frac{(\gamma_V^2 - 1) \cdot R \cdot \cos(\omega \cdot t_2)}{\gamma_V \cdot V} + (\gamma_V \cdot t_2) \tag{173}$$

- relationships between the projections v_{x11} and v_{y11} of speed v_{11} of the body 1 at time t_1 in the stationary reference system $O_1x_1y_1z_1$ and similar projections v_{x21} and v_{y21} of speed v_{21} of the body 1 in the mobile reference system $O_2x_2y_2z_2$ at time t_2 , which corresponds to the time t_1 in the stationary reference system $O_1x_1y_1z_1$:

$$v_{x11} = \frac{v_{x21} + V}{\frac{(\gamma_V^2 - 1) \cdot v_{x21}}{\gamma_V^2 \cdot V} + 1} \tag{174}$$

$$v_{y11} = \frac{v_{y21}}{\frac{(\gamma_V^2 - 1) \cdot v_{x21}}{\gamma_V \cdot V} + \gamma_V} \tag{175}$$

Similarly, for the consideration of motion of the body 2 can be written:

- relationships between coordinates x_{12} and y_{12} of the body 2 at time t_1 in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{22} and y_{22} of the body 2 in the mobile reference system $O_2x_2y_2z_2$ at time t_2 , which corresponds to the time t_1 in the stationary reference system $O_1x_1y_1z_1$:

$$x_{12} = \gamma_V \cdot [x_{22} + (V \cdot t_2)] \tag{176}$$

$$x_{22} = \gamma_V \cdot [x_{12} - (V \cdot t_1)] \tag{177}$$

$$y_{12} = y_{22} \tag{89}$$

- relationship between the values of times t_1 and t_2 in describing the motion of the body 2:

$$t_1 = \frac{(\gamma_V^2 - 1) \cdot x_{22}}{\gamma_V \cdot V} + (\gamma_V \cdot t_2) \qquad (178)$$

$$t_2 = \frac{(1 - \gamma_V^2) \cdot x_{12}}{\gamma_V \cdot V} + (\gamma_V \cdot t_1) \qquad (179)$$

while taking into account the equation (159) formula (178) becomes:

$$t_1 = -\frac{(\gamma_V^2 - 1) \cdot R \cdot \cos(\omega \cdot t_2)}{\gamma_V \cdot V} + (\gamma_V \cdot t_2) \qquad (180)$$

- relationships between the projections v_{x12} and v_{y12} of speed v_{12} of the body 2 at time t_1 in the stationary reference system $O_1x_1y_1z_1$ and similar projections v_{x22} and v_{y22} of speed v_{22} of the body 2 in the mobile reference system $O_2x_2y_2z_2$ at time t_2 , which corresponds to the time t_1 in the stationary reference system $O_1x_1y_1z_1$:

$$v_{x12} = \frac{v_{x22} + V}{\frac{(\gamma_V^2 - 1) \cdot v_{x22}}{\gamma_V^2 \cdot V} + 1} \qquad (181)$$

$$v_{y12} = \frac{v_{y22}}{\frac{(\gamma_V^2 - 1) \cdot v_{x22}}{\gamma_V \cdot V} + \gamma_V} \qquad (182)$$

Also to consider the motion of *i*-the point of the string 3, which is located at a distance ρ_i from point O on the segment from point O to the body 1 in the mobile reference system $O_2x_2y_2z_2$, we can write the following:

- relationships between coordinates $x_{11\rho i}$ and $y_{11\rho i}$ of *i*-the point of the string 3 at time t_1 in the stationary reference system $O_1x_1y_1z_1$ and coordinates $x_{21\rho i}$ and $y_{21\rho i}$ of *i*-the point of the string 3 in the mobile reference system $O_2x_2y_2z_2$ at time t_2 , which corresponds to the time t_1 in the stationary reference system $O_1x_1y_1z_1$:

$$x_{11\rho i} = \gamma_V \cdot \left[x_{21\rho i} + (V \cdot t_2)\right] \qquad (183)$$

$$x_{21\rho i} = \gamma_V \cdot \left[x_{11\rho i} - (V \cdot t_1)\right] \qquad (184)$$

$$y_{11\rho i} = y_{21\rho i} \qquad (185)$$

- relationship between the values of times t_1 and t_2 in describing the motion of *i*-the point of the string 3:

$$t_1 = \frac{(\gamma_V^2 - 1) \cdot x_{21\rho i}}{\gamma_V \cdot V} + (\gamma_V \cdot t_2) \tag{186}$$

$$t_2 = \frac{(1 - \gamma_V^2) \cdot x_{11\rho i}}{\gamma_V \cdot V} + (\gamma_V \cdot t_1) \tag{187}$$

while taking into account the equation (165) formula (186) becomes:

$$t_1 = \frac{(\gamma_V^2 - 1) \cdot \rho_i \cdot \cos(\omega \cdot t_2)}{\gamma_V \cdot V} + (\gamma_V \cdot t_2) \tag{188}$$

- relationships between the projections $v_{x11\rho i}$ and $v_{y11\rho i}$ of speed $v_{11\rho i}$ of *i*-the point of the string 3 at time t_1 in the stationary reference system $O_1x_1y_1z_1$ and similar projections $v_{x21\rho i}$ and $v_{y21\rho i}$ of speed $v_{21\rho i}$ of *i*-the point of the string 3 in the mobile reference system $O_2x_2y_2z_2$ at time t_2, which corresponds to the time t_1 in the stationary reference system $O_1x_1y_1z_1$:

$$v_{x11\rho i} = \frac{v_{x21\rho i} + V}{\frac{(\gamma_V^2 - 1) \cdot v_{x21\rho i}}{\gamma_V^2 \cdot V} + 1} \tag{189}$$

$$v_{y11\rho i} = \frac{v_{y21\rho i}}{\frac{(\gamma_V^2 - 1) \cdot v_{x21\rho i}}{\gamma_V \cdot V} + \gamma_V} \tag{190}$$

Also to consider the motion of *j*-the point of the string 3, which is located at a distance ρ_j from point O on the segment from point O to the body 2 in the mobile reference system $O_2x_2y_2z_2$, we can write the following:

- relationships between coordinates $x_{12\rho j}$ and $y_{12\rho j}$ of *j*-the point of the string 3 at time t_1 in the stationary reference system $O_1x_1y_1z_1$ and coordinates $x_{22\rho j}$ and $y_{22\rho j}$ of *j*-the point of the string 3 in the mobile reference system $O_2x_2y_2z_2$ at time t_2, which corresponds to the time t_1 in the stationary reference system $O_1x_1y_1z_1$:

$$x_{12\rho j} = \gamma_V \cdot [x_{22\rho j} + (V \cdot t_2)] \tag{191}$$

$$x_{22\rho j} = \gamma_V \cdot [x_{12\rho j} - (V \cdot t_1)] \tag{192}$$

$$y_{12\rho j} = y_{22\rho j} \tag{193}$$

- relationship between the values of times t_1 and t_2 in describing the motion of *j*-the point of the string 3:

$$t_1 = \frac{(\gamma_V^2 - 1) \cdot x_{22\rho j}}{\gamma_V \cdot V} + (\gamma_V \cdot t_2) \qquad (194)$$

$$t_2 = \frac{(1 - \gamma_V^2) \cdot x_{12\rho j}}{\gamma_V \cdot V} + (\gamma_V \cdot t_1) \qquad (195)$$

while taking into account the equation (167) formula (194) becomes:

$$t_1 = -\frac{(\gamma_V^2 - 1) \cdot \rho_j \cdot \cos(\omega \cdot t_2)}{\gamma_V \cdot V} + (\gamma_V \cdot t_2) \qquad (196)$$

- relationships between the projections $v_{x12\rho j}$ and $v_{y12\rho j}$ of speed $v_{12\rho j}$ of j-the point of the string 3 at time t_1 in the stationary reference system $O_1x_1y_1z_1$ and similar projections $v_{x22\rho j}$ and $v_{y22\rho j}$ of speed $v_{22\rho j}$ of j-the point of the string 3 in the mobile reference system $O_2x_2y_2z_2$ at time t_2, which corresponds to the time t_1 in the stationary reference system $O_1x_1y_1z_1$:

$$v_{x12\rho j} = \frac{v_{x22\rho j} + V}{\frac{(\gamma_V^2 - 1) \cdot v_{x22\rho j}}{\gamma_V^2 \cdot V} + 1} \qquad (197)$$

$$v_{y12\rho j} = \frac{v_{y22\rho j}}{\frac{(\gamma_V^2 - 1) \cdot v_{x22\rho j}}{\gamma_V \cdot V} + \gamma_V} \qquad (198)$$

In order to initiate testing of the law of conservation of momentum must select two points in time in the stationary inertial reference system $O_1x_1y_1z_1$.

First time - this is t_{1p} .

Suppose that in the stationary reference system $O_1x_1y_1z_1$ at time t_1 , equal to t_{1p} , the bodies 1 and 2 are on the line parallel to the axis O_1y_1 (or coinciding with it), ie where:

$$x_{11} = x_{12} \qquad (199)$$

Condition (199) is possible only in the case, when in the mobile reference system $O_2x_2y_2z_2$ at the time t_2 , equal to t_{2p} , corresponding to the time t_{1p} in the stationary reference system $O_1x_1y_1z_1$, the following conditions:

$$x_{21} = x_{22} \qquad (200)$$

$$\omega \cdot t_{2p} = \frac{\pi}{2} \qquad (201)$$

As shown in Fig.15, according to equations (201), (153) - (156) in the mobile reference system $O_2x_2y_2z_2$ at time t_{2p} the bodies 1 and 2, respectively, have the following values of the projections $v_{21\,xp}$, $v_{21\,yp}$ and $v_{22\,xp}$, $v_{22\,yp}$ of speeds of his movement on the axis O_2x_2 and O_2y_2 :

$$v_{21\,xp} = -\,v_R \tag{99}$$

$$v_{21\,yp} = 0 \tag{100}$$

$$v_{22\,xp} = v_R \tag{101}$$

$$v_{22\,yp} = 0 \tag{102}$$

And in accordance with equations (201), (161)-(164) in the mobile reference system $O_2x_2y_2z_2$ at time t_{2p} the *i*-the point of the string 3, which is located at a distance ρ_i from point O on the segment from point O to the body 1, and the *j*-the point of the string 3, which is located at a distance ρ_j from point O on the segment from point O to the body 2, respectively, have the following values of the projections $v_{21x\rho ip}$, $v_{21y\rho ip}$ and $v_{22x\rho jp}$, $v_{22y\rho jp}$ of speeds of his movement on the axis O_2x_2 and O_2y_2 :

$$v_{21\,x\rho i\,\mathrm{p}} = -(v_R \;\cdot\; \frac{\rho_i}{R}) \tag{202}$$

$$v_{21\,y\rho i\,\mathrm{p}} = 0 \tag{203}$$

$$v_{22\,x\rho j\,\mathrm{p}} = v_R \;\cdot\; \frac{\rho_j}{R} \tag{204}$$

$$v_{22\,y\rho j\,\mathrm{p}} = 0 \tag{205}$$

A second point in time we choose t_{1h} .

Suppose that, as shown in Fig.16, in the stationary reference system $O_1x_1y_1z_1$ at time t_1, equal to t_{1h} , the position of body 1 will be consistent the position of the body 1 at time t_2 , equal to t_{21h} :

$$t_{21\,h} \;=\; 0 \tag{206}$$

in the mobile reference system $O_2x_2y_2z_2$, ie when the body 1 will be on the axis O_2x_2 .

The value of time t_{1h} can be determined from equation (173) on the basis of conditions (206):

$$t_{1h} = \frac{(\gamma_V^2 - 1) \cdot R}{\gamma_V \cdot V} \quad (207)$$

According to equations (206), (153), (154) in the mobile reference system $O_2x_2y_2z_2$ at time t_{21h} body 1 will have the following values of the projections v_{21xh} and v_{21yh} of the speed of his movement on the axis O_2x_2 and O_2y_2 :

$$v_{21xh} = 0 \quad (111)$$

$$v_{21yh} = -v_R \quad (112)$$

In the stationary reference system $O_1x_1y_1z_1$ at the time t_1, equal to t_{1h} , the position of body 2 will be consistent the position of the body 2 in the mobile reference system $O_2x_2y_2z_2$ at time t_2 , equal to t_{22h} , which can be determined on the basis of equations (180) and (207):

$$\frac{(\gamma_V^2 - 1) \cdot R}{\gamma_V \cdot V} = -\frac{(\gamma_V^2 - 1) \cdot R \cdot \cos(\omega \cdot t_{22h})}{\gamma_V \cdot V} + (\gamma_V \cdot t_{22h}) \quad (208)$$

or:

$$(\omega \cdot t_{22h}) = \frac{(\gamma_V^2 - 1) \cdot [1 + \cos(\omega \cdot t_{22h})] \cdot v_R}{\gamma_V^2 \cdot V} \quad (209)$$

Similar in the stationary reference system $O_1x_1y_1z_1$ at the time t_1, equal to t_{1h} , the position of the *i*-the point of the string 3 will be consistent the position of the *i*-the point of the string 3, which is located at a distance ρ_i from point O on the segment from point O to the body 1, in the mobile reference system $O_2x_2y_2z_2$ at time t_2 , equal to $t_{21\rho i h}$, which can be determined on the basis of equations (188) and (207):

$$\frac{(\gamma_V^2 - 1) \cdot R}{\gamma_V \cdot V} = \frac{(\gamma_V^2 - 1) \cdot \rho_i \cdot \cos(\omega \cdot t_{21\rho i h})}{\gamma_V \cdot V} + (\gamma_V \cdot t_{21\rho i h}) \quad (210)$$

or:

$$(\omega \cdot t_{21\rho i h}) = \frac{(\gamma_V^2 - 1) \cdot v_R}{\gamma_V \cdot V} \cdot \left\{1 - \left[\frac{\rho_i}{R} \cdot \cos(\omega \cdot t_{21\rho i h})\right]\right\} \quad (211)$$

Also in the stationary reference system $O_1x_1y_1z_1$ at the time t_1, equal to t_{1h} , the position of the *j*-the point of the string 3 will be consistent the position of the *j*-the point of the string 3, which is located at a distance ρ_j from point O on the segment from point O to the body 2, in the mobile reference system $O_2x_2y_2z_2$ at time t_2 , equal to $t_{22\rho jh}$, which can be determined on the basis of equations (196) and (207):

$$\frac{(\gamma_V^2-1)\cdot R}{\gamma_V\ \cdot V} = -\ \frac{(\gamma_V^2-1)\cdot\rho_j\ \cdot\cos\left(\omega\cdot t_{22\rho jh}\right)}{\gamma_V\ \cdot V}+\left(\gamma_V\ \cdot t_{22\rho jh}\right) \quad (212)$$

or:

$$\left(\omega\cdot t_{22\rho jh}\right) = \frac{(\gamma_V^2-1)\cdot\ v_R}{\gamma_V^2\cdot\ V}\cdot\left\{1+\left[\frac{\rho_j}{R}\cdot\cos\left(\omega\ \cdot\ t_{22\rho jh}\right)\right]\right\} \quad (213)$$

To handle complex calculations in equations (209), (211) and (213), the values of momentums will try to determine by simple numerical examples.

For consideration in the mobile reference system $O_2x_2y_2z_2$ the string 3 conditionally divided by 17 equal parts (i = 0, 1, 2, 3, 4, 5, 6, 7, 8 and j = 1, 2, 3, 4, 5, 6, 7, 8) with accommodation in the center of each part the point of the body with rest mass m_{017} , equal to:

$$m_{017} = \frac{m_0}{17} \quad (214)$$

In this case, the distance ρ_i from point O to the i-the point of the string 3, located on the segment from point O to the body 1, will be equal to:

$$\rho_i = \frac{2\ \cdot\ i}{17} \quad (215)$$

And the distance ρ_j from point O to the j-the point of the string 3, located on the segment from point O to the body 2, will be equal to:

$$\rho_j = \frac{2\ \cdot\ j}{17} \quad (216)$$

First, consider the case, when the values of the coefficient of proportionality γ_V (also γ_v) lie in the range $\gamma_V \geq 1$ (and $\gamma_v \geq 1$).

In the case, if the values of the coefficient of proportionality γ_v are in the range $\gamma_v \geq 1$, then as follows from formula (72), in any inertial reference system $Oxyz$ the projections P_x and P_y of the momentum of a material point, moving with the speed v and having a rest mass m_o, on the axis Ox and Oy , respectively, can be written:

$$P_x = \frac{m_o\, v_x}{\sqrt{1 - \frac{\left(v_x^2 + v_y^2\right)}{c_1^2}}} \qquad (217)$$

$$P_y = \frac{m_o\, v_y}{\sqrt{1 - \frac{\left(v_x^2 + v_y^2\right)}{c_1^2}}} \qquad (218)$$

where: v_x and v_y - the projections of the speed v of a material point on the axis Ox and Oy , respectively.

Assume in the considered example 3 (shown in Fig.13 - Fig.17), that:

$$\frac{V}{c_1} = 0{,}9 \qquad (219)$$

$$\frac{v_R}{c_1} = 0{,}8 \qquad (220)$$

$$\frac{m_0}{M_0} = 0{,}1 \qquad (221)$$

To determine the values of the momentums of the system of bodies 1 and 2 and string 3 in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} will use equation (99) - (102), (217), (218), the raw data (214) - (216), (219) - (221) and the formulas, derived from the equations (174), (175), (181), (182), (189), (190), (197) and (198), taking into account equation (33) :

$$v_{x11} = \frac{v_{x21} + V}{1 + \frac{V \cdot v_{x21}}{c_1^2}} \qquad (222)$$

$$v_{y11} = \frac{v_{y21} \cdot \sqrt{1 - \frac{V^2}{c_1^2}}}{1 + \frac{V \cdot v_{x21}}{c_1^2}} \qquad (223)$$

$$v_{x12} = \frac{v_{x22} + V}{1 + \frac{V \cdot v_{x22}}{c_1^2}} \qquad (224)$$

$$v_{y12} = \frac{v_{y22} \cdot \sqrt{1 - \frac{V^2}{c_1^2}}}{1 + \frac{V \cdot v_{x22}}{c_1^2}} \qquad (225)$$

$$v_{x11\rho i} = \frac{v_{x21\rho i} + V}{1 + \frac{V \cdot v_{x21\rho i}}{c_1^2}} \qquad (226)$$

$$v_{y11\rho i} = \frac{v_{y21\rho i} \cdot \sqrt{1 - \frac{V^2}{c_1^2}}}{1 + \frac{V \cdot v_{x21\rho i}}{c_1^2}} \qquad (227)$$

$$v_{x12\rho j} = \frac{v_{x22\rho j} + V}{1 + \frac{V \cdot v_{x22\rho j}}{c_1^2}} \qquad (228)$$

$$v_{y12\rho j} = \frac{v_{y22\rho j} \cdot \sqrt{1 - \frac{V^2}{c_1^2}}}{1 + \frac{V \cdot v_{x22\rho j}}{c_1^2}} \qquad (229)$$

The results of digital calculations presented in Tab.7.

Tab.7

Range $\gamma_V \geq 1$ (and $\gamma_v \geq 1$). Time t_{1p} .

Object	Mobile reference system $O_2x_2y_2z_2$		Stationary reference system $O_1x_1y_1z_1$			
	The projections of the velocity (the dimension c_1)		The projections of the velocity (the dimension c_1)		Projections of the momentum (the dimension $c_1 \cdot M_0$)	
	on the axis O_2x_2	on the axis O_2y_2	on the axis O_1x_1	on the axis O_1y_1	on the axis O_1x_1	on the axis O_1y_1
Body 1	-0,8	0	0,3571429	0	0,3823596	0
Body 2	0,8	0	0,9883721	0	6,5001125	0
Bodies 1 and 2					6,8824472	0
$i = 0$	0	0	0,9	0	0,0121455	0
$i = 1$	-0,09412	0	0,8804627	0	0,0109239	0
$i = 2$	-0,18824	0	0,8569405	0	0,0097801	0
$i = 3$	-0,28235	0	0,8280757	0	0,0086887	0
$i = 4$	-0,37647	0	0,7918149	0	0,0076261	0
$i = 5$	-0,47059	0	0,744898	0	0,0065676	0
$i = 6$	-0,56471	0	0,6818182	0	0,0054827	0
$i = 7$	-0,65882	0	0,5924855	0	0,0043263	0
$i = 8$	-0,75294	0	0,4562044	0	0,0030156	0
$j = 1$	0,094118	0	0,9164859	0	0,0134755	0
$j = 2$	0,188235	0	0,9305835	0	0,0149531	0
$j = 3$	0,282353	0	0,99427767	0	0,0166327	0
$j = 4$	0,376471	0	0,9534271	0	0,0185940	0
$j = 5$	0,470588	0	0,9628099	0	0,0209623	0
$j = 6$	0,564706	0	0,9711388	0	0,0239506	0
$j = 7$	0,658824	0	0,978582	0	0,0279629	0
$j = 8$	0,752941	0	0,9852735	0	0,0338959	0
String 3					0,2389836	0
Bodies 1 and 2 and string 3					7,1214557	0

To determine the values of the momentums of the system of bodies 1 and 2 and string 3 in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} will use equation (155)-(156), (161)-(164), (111)-(112), (217)-(218), (222)-(229), the raw data (219)-(221), (214)-(216) and the formulas, derived from the equations (209), (211) and (213), taking into account equation (33) :

$$(\omega \cdot t_{22h}) = \frac{V \cdot v_R \cdot [1 + \cos(\omega \cdot t_{22h})]}{c_1^2} \tag{230}$$

$$(\omega \cdot t_{21\rho i h}) = \frac{V \cdot v_R}{c_1^2} \cdot \left\{1 - \left[\frac{\rho_i}{R} \cdot \cos(\omega \cdot t_{21\rho i h})\right]\right\} \tag{231}$$

$$(\omega \cdot t_{22\rho j h}) = \frac{V \cdot v_R}{c_1^2} \cdot \left\{1 + \left[\frac{\rho_j}{R} \cdot \cos(\omega \cdot t_{22\rho j h})\right]\right\} \tag{232}$$

The results of digital calculations presented in Tab.8.

Tab.8

Range $\gamma_V \geq 1$ (and $\gamma_v \geq 1$). Time t_{1h} .

Object	Mobile reference system $O_2x_2y_2z_2$		Stationary reference system $O_1x_1y_1z_1$			
	The projections of the velocity (the dimension c_1)		The projections of the velocity (the dimension c_1)		Projections of the momentum (the dimension $c_1 \cdot M_0$)	
	on the axis O_2x_2	on the axis O_2y_2	on the axis O_1x_1	on the axis O_1y_1	on the axis O_1x_1	on the axis O_1y_1
Body 1	0	-0,8	0,9	-0,34871	3,441236	-1,333333
Body 2	0,700743	0,385953	0,9816482	0,103168	6,1205934	0,6432543
Bodies 1 and 2					9,5618294	-0,690079
$i = 0$	0	0	0,9	0	0,0121455	0
$i = 1$	-0,05716	-0,07477	0,8885503	-0,03436	0,0114249	-0,000442
$i = 2$	-0,10286	-0,15765	0,8784626	-0,07573	0,0109532	-0,000944
$i = 3$	-0,13452	-0,24825	0,8709212	-0,12311	0,0107684	-0,001522
$i = 4$	-0,14977	-0,3454	0,8671108	-0,17401	0,0109284	-0,002193
$i = 5$	-0,14699	-0,44704	0,8678151	-0,22457	0,0115169	-0,002980
$i = 6$	-0,12573	-0,55053	0,8730642	-0,27059	0,0126608	-0,003924
$i = 7$	-0,08687	-0,65307	0,8820957	-0,30881	0,0145863	-0,005106
$i = 8$	-0,03235	-0,75225	0,8936691	-0,33773	0,0177924	-0,006724
$j= 1$	0,066205	0,066896	0,9118716	0,027519	0,013097	0,000395
$j= 2$	0,139393	0,1265	0,9235324	0,048994	0,014282	0,000758
$j= 3$	0,217908	0,179553	0,934614	0,065433	0,0157261	0,001101
$j= 4$	0,300464	0,22683	0,9449365	0,077827	0,0174868	0,001440
$j= 5$	0,386083	0,269061	0,9544394	0,087037	0,0196698	0,001794
$j= 6$	0,474026	0,306907	0,9631315	0,093772	0,0224678	0,002187
$j= 7$	0,563739	0,34095	0,971058	0,098594	0,0262572	0,002666
$j= 8$	0,654805	0,371687	0,9782804	0,101939	0,0318835	0,003322
String 3					0,2736473	-0,010172
Bodies 1 and 2 and string 3					9,8354767	-0,700351

As a result of numerical calculation for the case, when the values of the coefficient of proportionality γ_V (also γ_v) lie in the range $\gamma_V \geq 1$ (and $\gamma_v \geq 1$) , it was found that, in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} the closed system of bodies 1 and 2 and string 3 has the projection of the momentum on the axis O_1x_1 , equal to $\mathbf{7{,}1214557} \cdot \boldsymbol{c_1} \cdot \boldsymbol{M_0}$, and the projection of the momentum on the axis O_1y_1 , equal to $\mathbf{0}$.

And in a stationary reference system $O_1x_1y_1z_1$ at time t_{1h} the closed system of bodies 1 and 2 and string 3 has the projection of the momentum on the axis O_1x_1 , equal to $\mathbf{9{,}8354767} \cdot \boldsymbol{c_1} \cdot \boldsymbol{M_0}$, and the projection of the momentum on the axis O_1y_1 , equal to $\mathbf{-0{,}700351} \cdot \boldsymbol{c_1} \cdot \boldsymbol{M_0}$.

As a result, we have a violation of the law of conservation of momentum for a closed mechanical system of bodies, because 7,1214557 ≠ 9,8354767 and 0 ≠ -0,700351.

Moreover, integration of mass of string 3 in calculating the momentum of the system of bodies 1 and 2 and string 3 leads to the aggravation of violating the law of conservation of momentum.

In the stationary reference system $O_1x_1y_1z_1$ in the case, when the values of the coefficient of proportionality γ_V (also γ_v) lie in the range $\gamma_V \geq 1$ (and $\gamma_v \geq 1$) , the momentum of a closed system of bodies 1 and 2 and string 3 is not constant, as is a function of time t_1 .

Next, consider the case, when the values of the coefficient of proportionality γ_V (also γ_v) lie in the range $0 < \gamma_V \leq 1$ (and $0 < \gamma_v \leq 1$).

In the case, if the values of the coefficient of proportionality γ_v are in the range $0 < \gamma_v \leq 1$, then as follows from formula (75), in any inertial reference system *Oxyz* the projections P_x and P_y of the momentum of a material point, moving with the speed v and having a rest mass m_o , on the axis *Ox* and *Oy*, respectively, can be written:

$$P_x = \frac{m_o\, v_x}{\sqrt{1 + \frac{\left(v_x^2 + v_y^2\right)}{c_2^2}}} \qquad (233)$$

$$P_y = \frac{m_o\, v_y}{\sqrt{1 + \frac{\left(v_x^2 + v_y^2\right)}{c_2^2}}} \qquad (234)$$

where: v_x and v_y - the projections of the speed v of a material point on the axis *Ox* and *Oy* , respectively.

Assume in the considered example 3 (shown in Fig.13 - Fig.17), that:

$$\frac{V}{c_2} = 0{,}9 \qquad (235)$$

$$\frac{v_R}{c_2} = 0{,}8 \qquad (236)$$

$$\frac{m_0}{M_0} = 0{,}1 \qquad (237)$$

To determine the values of the momentums of the system of bodies 1 and 2 and string 3 in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} will use equation (99) - (102), (233)-(234), the raw data (214) - (216), (235)-(237) and the formulas, derived from the equations (174), (175), (181), (182), (189), (190), (197) and (198), taking into account equation (34) :

$$v_{x11} = \frac{v_{x21} + V}{1 - \frac{V \cdot v_{x21}}{c_2^2}} \qquad (238)$$

$$v_{y11} = \frac{v_{y21} \cdot \sqrt{1 + \frac{V^2}{c_2^2}}}{1 - \frac{V \cdot v_{x21}}{c_2^2}} \qquad (239)$$

$$v_{x12} = \frac{v_{x22} + V}{1 - \frac{V \cdot v_{x22}}{c_2^2}} \qquad (240)$$

$$v_{y12} = \frac{v_{y22} \cdot \sqrt{1 + \frac{V^2}{c_2^2}}}{1 - \frac{V \cdot v_{x22}}{c_2^2}} \qquad (241)$$

$$v_{x11\rho i} = \frac{v_{x21\rho i} + V}{1 - \frac{V \cdot v_{x21\rho i}}{c_2^2}} \qquad (242)$$

$$v_{y11\rho i} = \frac{v_{y21\rho i} \cdot \sqrt{1 + \frac{V^2}{c_2^2}}}{1 - \frac{V \cdot v_{x21\rho i}}{c_2^2}} \qquad (243)$$

$$v_{x12\rho j} = \frac{v_{x22\rho j} + V}{1 - \frac{V \cdot v_{x22\rho j}}{c_2^2}} \qquad (244)$$

$$v_{y12\rho j} = \frac{v_{y22\rho j} \cdot \sqrt{1 + \frac{V^2}{c_2^2}}}{1 - \frac{V \cdot v_{x22\rho j}}{c_2^2}} \qquad (245)$$

The results of digital calculations presented in Tab.9.

Tab.9

Range $0 < \gamma_V \leq 1$ (and $0 < \gamma_v \leq 1$) . Time t_{1p} .

Object	Mobile reference system отсчета $O_2x_2y_2z_2$		Stationary reference system $O_1x_1y_1z_1$			
	The projections of the velocity (the dimension c_2)		The projections of the velocity (the dimension c_2)		Projections of the momentum (the dimension $c_2 \cdot M_0$)	
	on the axis O_2x_2	on the axis O_2y_2	on the axis O_1x_1	on the axis O_1y_1	on the axis O_1x_1	on the axis O_1y_1
Body 1	-0,8	0	0,0581395	0	0,0580415	0
Body 2	0,8	0	6,0714286	0	0,9867059	0
Bodies 1 and 2					1,0447474	0
$i = 0$	0	0	0,9	0	0,0039351	0
$i = 1$	-0,09412	0	0,7429501	0	0,0035081	0
$i = 2$	-0,18824	0	0,6086519	0	0,0030583	0
$i = 3$	-0,28235	0	0,4924953	0	0,0025989	0
$i = 4$	-0,37647	0	0,3910369	0	0,0021422	0
$i = 5$	-0,47059	0	0,3016529	0	0,0016988	0
$i = 6$	-0,56471	0	0,2223089	0	0,0012765	0
$i = 7$	-0,65882	0	0,1514032	0	0,0008806	0
$i = 8$	-0,75294	0	0,00876578	0	0,0005137	0
$j= 1$	0,094118	0	1,0861183	0	0,0043275	0
$j= 2$	0,188235	0	1,3101983	0	0,004676	0
$j= 3$	0,282353	0	1,5851735	0	0,0049751	0
$j= 4$	0,376471	0	1,930605	0	0,0052232	0
$j= 5$	0,470588	0	2,377551	0	0,0054223	0
$j= 6$	0,564706	0	2,9784689	0	0,0055764	0
$j= 7$	0,658824	0	3,8294798	0	0,0056915	0
$j= 8$	0,752941	0	5,1277372	0	0,0057736	0
String 3					0,0612779	0
Bodies 1 and 2 and string 3					1,106025	0

To determine the values of the momentums of the system of bodies 1 and 2 and string 3 in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} will use equation (155)-(156), (161)-(164), (111)-(112), (233)-(234), (238)-(245), the raw data (235)-(237), (214)-(216) and the formulas, derived from the equations (209), (211) and (213), taking into account equation (34) :

$$(\omega \cdot t_{22h}) \;=\; -\frac{V \cdot v_R \cdot [1 + \cos(\omega \cdot t_{22h})]}{c_2^2} \qquad (246)$$

$$\left(\omega \cdot t_{21\rho i h}\right) \;=\; -\frac{V \cdot v_R}{c_2^2} \cdot \left\{1 - \left[\frac{\rho_i}{R} \cdot \cos\left(\omega \cdot t_{21\rho i h}\right)\right]\right\} \qquad (247)$$

$$\left(\omega \cdot t_{22\rho j h}\right) \;=\; -\frac{V \cdot v_R}{c_2^2} \cdot \left\{1 + \left[\frac{\rho_j}{R} \cdot \cos\left(\omega \cdot t_{22\rho j h}\right)\right]\right\} \qquad (248)$$

The results of digital calculations presented in Tab.10.

Tab.10

Range $0 < \gamma_V \leq 1$ (and $0 < \gamma_v \leq 1$) . Time t_{1h} .

Object	Mobile reference system $O_2x_2y_2z_2$		Stationary reference system $O_1x_1y_1z_1$			
	The projections of the velocity (the dimension c_2)		The projections of the velocity (the dimension c_2)		Projections of the momentum (the dimension $c_2 \cdot M_0$)	
	on the axis O_2x_2	on the axis O_2y_2	on the axis O_1x_1	on the axis O_1y_1	on the axis O_1x_1	on the axis O_1y_1
Body 1	0	-0,8	0,9	-1,07629	0,5223737	-0,624695
Body 2	-0,700743	0,385953	0,1221935	0,318425	0,1156519	0,3013783
Bodies 1 and 2					0,6380255	-0,323317
$i = 0$	0	0	0,9	0	0,0039351	0
$i = 1$	0,05716	-0,07477	1,0090732	-0,10605	0,0041666	-0,000438
$i = 2$	0,10286	-0,15765	1,1051724	-0,23373	0,0043091	-0,000911
$i = 3$	0,13452	-0,24825	1,177014	-0,37999	0,004353	-0,001405
$i = 4$	0,14977	-0,3454	1,2133127	-0,53708	0,0042956	-0,001901
$i = 5$	0,14699	-0,44704	1,2066039	-0,69313	0,004142	-0,002379
$i = 6$	0,12573	-0,55053	1,1565986	-0,83517	0,0039051	-0,00282
$i = 7$	0,08687	-0,65307	1,0705621	-0,95313	0,0036032	-0,003208
$i = 8$	0,03235	-0,75225	0,9603103	-1,04239	0,0032566	-0,003535
$j = 1$	-0,066205	0,066896	0,7869078	0,084938	0,0036296	0,000392
$j = 2$	-0,139393	0,1265	0,6758229	0,151217	0,0032682	0,000731
$j = 3$	-0,217908	0,179553	0,5702556	0,201957	0,0028701	0,001016
$j = 4$	-0,300464	0,22683	0,4719207	0,240211	0,0024533	0,001249
$j = 5$	-0,386083	0,269061	0,3813932	0,268639	0,0020331	0,001432
$j = 6$	-0,474026	0,306907	0,2985891	0,289426	0,0016218	0,001572
$j = 7$	-0,563739	0,34095	0,2230786	0,304306	0,0012277	0,001675
$j = 8$	-0,654805	0,371687	0,1542765	0,314633	0,0008564	0,001747
String 3					0,0539268	-0,006784
Bodies 1 and 2 and string 3					0,6919523	-0,330101

As a result of numerical calculation for the case, when the values of the coefficient of proportionality γ_V (also γ_v) lie in the range $0 < \gamma_V < 1$ (and

$0 < \gamma_v < 1$), it was found, that in the stationary reference system $O_1x_1y_1z_1$ at time t_{1p} the closed system of bodies 1 and 2 and string 3 has the projection of the momentum on the axis O_1x_1, equal to $\mathbf{1{,}106025 \cdot c_2 \cdot M_0}$, and the projection of the momentum on the axis O_1y_1, equal to $\mathbf{0}$.

And in a stationary reference system $O_1x_1y_1z_1$ at time t_{1h} the closed system of bodies 1 and 2 and string 3 has the projection of the momentum on the axis O_1x_1, equal to $\mathbf{0{,}6919523 \cdot c_2 \cdot M_0}$, and the projection of the momentum on the axis O_1y_1, equal to $\mathbf{-0{,}330101 \cdot c_2 \cdot M_0}$.

As a result, we have a violation of the law of conservation of momentum for a closed mechanical system of bodies, because $1{,}106025 \neq 0{,}6919523$ and $0 \neq -0{,}330101$.

Moreover, integration of mass of string 3 in calculating the momentum of the system of bodies 1 and 2 and string 3 also leads to the aggravation of violating the law of conservation of momentum.

In the stationary reference system $O_1x_1y_1z_1$ in the case, when the values of the coefficient of proportionality γ_V (also γ_v) lie in the range $0 < \gamma_V < 1$ (and $0 < \gamma_v < 1$), the momentum of a closed system of bodies 1 and 2 and string 3 is also not constant, as is a function of time t_1.

9. Conclusion

In conclusion, we note the following:

- there are possible two variants of the relationship between coordinates and time in inertial reference systems for values of the coefficient of proportionality γ_V, are in the range $\gamma_V > 1$ and $0 < \gamma_V < 1$.

- use of the special theory of relativity in dealing with individual examples (examples 2 and 3) may lead to non-compliance with the law of conservation of momentum for a closed mechanical system in the inertial reference systems.

Given, that the law of conservation of momentum associated with the homogeneity of space, we can assume, that the failure of the law of conservation of momentum will lead to non-compliance with conditions of symmetry of space and time, on which is based the special theory of relativity.

The results obtained, when considering the examples 2 and 3 show that, if true to the law of conservation of momentum, it is necessary to finalize the special theory of relativity, or if it is true the special theory of relativity, then, consequently, incorrect law of conservation of momentum - possible to change the momentum of a closed system over time in the inertial reference systems.

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