

Column: the mathematical physics.  
Subject-matter: the special theory of relativity

## **APPLICABILITY OF THE SPECIAL THEORY OF RELATIVITY FOR INERTIAL REFERENCE SYSTEMS BASED ON THE SYMMETRY OF SPACE AND TIME**

(additions and corrections article)

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This article seeks to determine whether the Lorentz transformation of the special theory of the relativity is the only possible relationship between the coordinates and time in inertial reference systems, as well as whether its findings are requirements imposed by the conditions of the symmetry of space and time.

### **I. The introduction**

Currently, the internet and various magazines contain numerous articles devoted to the criticism of the special theory of relativity.

You can also note that the criticism of the special theory of the relativity mainly consists of a description of the logical inconsistencies of its findings with respect to the real presentation of space and time. But the special theory is an idealized mathematical model, built under the certain conditions, and therefore the results may not be made available outside the conditions set for it.

In my opinion, if we criticize the special theory, the criticism would have to start with its mathematical model. Maybe it would be useful to re-examine this mathematical model and test its conclusions through the conditions that lay in its creation.

### I.1. The brief history of the creation of the special theory of relativity

At the turn of the XIX/XX century, the efforts of the greatest physicists of that time established the special theory of the relativity. At the end of XIX century, two of the most important sections of physics - mechanics and the electrodynamics displayed serious contradictions. Mechanics contained the Galilean principle of the relativity - full equality of reference systems, moving relative to one another. In electrodynamics, the fundamental place was held by the idea of an ether, which filled all of space and in which all physical processes took place, including electromagnetic fluctuations. This required that the movement of particles and fields should be described by coordinates tightly linked to the ether, which served as an absolute reference system.

In years 1881, 1886/1887 the Michelson-Morley experiments were unable to register an "ether wind". As a result, the ether theory of light, seemingly confirmed by experiments, was shown to be inconsistent with classical mechanics. In 1889, Irish physicist D. Fitzgerald proposed that the longitudinal  $l'$  of a body moving at the speed  $V$  through the ether, is reduced by:

$$l' = l \cdot \sqrt{1 - \frac{V^2}{c^2}} \quad (1)$$

where:  $c$  is the speed of the light,

$l$  - the fixed length of the body.

In 1892 the Dutch physicist, H. Lorentz, added to the D. Fitzgerald hypothesis, the idea of a "local" time  $t'$  associated with the "true" universal time  $t$  in the transformation:

$$t' = t - \left( \frac{\mathbf{x} \cdot \mathbf{v}}{c^2} \right) \quad (2)$$

where:  $\mathbf{v}$  is the speed of the movement of the body while passing the point of the space with the coordinate  $\mathbf{x}$ .

Also H. Lorentz modified the Galilean transformation for high speeds:

$$x_1 = \beta \cdot [x_2 + (V \cdot t_2)] \quad (3)$$

$$y_1 = y_2 \quad (4)$$

$$z_1 = z_2 \quad (5)$$

$$t_1 = \beta \cdot \left[ t_2 + \left( \frac{x_2 \cdot V}{c^2} \right) \right] \quad (6)$$

by introducing a "relativistic" factor  $\beta$  :

$$\beta = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (7)$$

Formulas (3) and (6) representing relations between inertial reference systems were named the Lorentz transformations.

As early as in 1881 the English physicist D. Thompson suggested that a mass  $\mathbf{M}$  moving with speed  $v$ , will be greater than mass  $\mathbf{M}_0$  in a state of rest, with the value  $\mathbf{M}$  being:

$$M = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (8)$$

## I.2. The special theory of the relativity

In 1905, A. Einstein considered the basic the fundamental principles of the two classic physical theories: from mechanics - the principle of the equality of all inertial reference systems (the principle of the relativity), and electrodynamics - the principle of the constancy of light speed.

The principle of the relativity states: **in the all inertial reference systems, all physical phenomena in the same state, operate the same way.** That is, the physical laws are independent (invariant) of the choice of the inertial reference system. Therefore, the equations expressing these laws have the same form in the all inertial reference systems.

The principle of the invariance of the speed of light: **the speed of the light in the vacuum is independent of the movement of the light source.** That is, the speed of light is the same in the all directions and in all inertial reference systems.

Using the principle of relativity and the principle of the constancy of the speed of the light, Einstein restated the Lorentz transformations, giving them a physical sense:

$$x_1 = \frac{x_2 + (V \cdot t_2)}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (9)$$

$$x_2 = \frac{x_1 - (V \cdot t_1)}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (10)$$

$$y_1 = y_2 \quad (11)$$

$$z_1 = z_2 \quad (12)$$

where:  $x_1, y_1, z_1$  – are the coordinates of the point **A** at time  $t_1$  in the fixed inertial reference system  $\mathbf{O}_1x_1y_1z_1$ ;

$x_2, y_2, z_2$  – the coordinates of point **A** at time  $t_2$  in the moving inertial reference system  $\mathbf{O}_2x_2y_2z_2$ , as shown in Fig. 1.

$$t_1 = \frac{t_2 + \left(\frac{x_2 \cdot V}{c^2}\right)}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (13)$$

$$t_2 = \frac{t_1 - \left(\frac{x_1 \cdot V}{c^2}\right)}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (14)$$

On the basis of formulas (9)-(14), the relationship between the projection  $v_{x2}, v_{y2}$  and  $v_{z2}$  of the speed at point **A** in the moving reference system  $\mathbf{O}_2x_2y_2z_2$  on the axis of the Cartesian coordinates, and the similar projection  $v_{x1}, v_{y1}$  and  $v_{z1}$  of the speed of the same point **A** in the fixed inertial reference system  $\mathbf{O}_1x_1y_1z_1$  was defined as:

$$v_{x1} = \frac{v_{x2} + V}{1 + \frac{V \cdot v_{x2}}{c^2}} \quad (15)$$

$$v_{x2} = \frac{v_{x1} - V}{1 - \frac{V \cdot v_{x1}}{c^2}} \quad (16)$$

$$v_{y1} = \frac{v_{y2} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V \cdot v_{x2}}{c^2}} \quad (17)$$

$$v_{y2} = \frac{v_{y1} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V \cdot v_{x1}}{c^2}} \quad (18)$$

$$v_{z1} = \frac{v_{z2} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V \cdot v_{x2}}{c^2}} \quad (19)$$

$$v_{z2} = \frac{v_{z1} \cdot \sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V \cdot v_{x1}}{c^2}} \quad (20)$$

According to the special theory of relativity the mass  $\mathbf{M}(\mathbf{V})$ , of the momentum  $\mathbf{P}(\mathbf{V})$ , and of the kinetic energy  $\mathbf{E}_k(\mathbf{V})$ , of the material point, moving at the speed  $\mathbf{V}$ , was expressed by the formulas:

$$\mathbf{M} = \frac{\mathbf{M}_0}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (21)$$

$$\mathbf{P}(\mathbf{V}) = \frac{\mathbf{M}_0 \cdot \mathbf{V}}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (22)$$

$$\mathbf{E}_k(\mathbf{V}) = \mathbf{M}_0 \cdot c^2 \cdot \left( \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} - 1 \right) \quad (23)$$

where:  $\mathbf{M}_0$  is the mass of the material point at rest.

Finally, it may be noted that the special theory of relativity was established primarily to explain the results of experiments (A. Michelson and others), leading to the question of the constancy of the speed of the light (or more precisely to the explanation of the constancy of the speed of the light).

## II. The kinematics

### II.1. "The special theory of relativity in general terms"

Here we relax the specific requirements of relativity to facilitate analysis.

Suppose that space is homogeneous and isotropic and time is homogeneous (that is, there is symmetry in space and the time).

We consider whether to use the principle of relativity: "in all inertial reference systems, all physical phenomena in the same state, operate in the same way."

In the absence of the need not to apply the principle of the invariance of the speed of light (that is, less stringent conditions apply),

suppose that there are two inertial reference system: fixed  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and moving  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , shown in Fig. 1 and that:

- the axes of the Cartesian coordinates of the systems  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  are parallel and equally directed;
- system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  is moving in system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  with constant speed  $V_2$  on the  $\mathbf{Ox}_1$  axis;
- a starting time ( $t_1=0$  and  $t_2=0$ ) in both systems is selected when coordinate centres  $\mathbf{O}_1$  and  $\mathbf{O}_2$  of the systems match.

Based on the symmetry of space and time, the relationship between the time and the coordinates of the same events in the two inertial reference systems: fixed  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and moving  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  can be written as follows:

$$x_1 = \beta_1 \cdot [x_2 + (V_1 \cdot t_2)] \quad (24)$$

$$x_2 = \beta_2 \cdot [x_1 + (V_2 \cdot t_1)] \quad (25)$$

$$y_1 = \beta_3 \cdot y_2 \quad (26)$$

$$y_2 = \beta_4 \cdot y_1 \quad (27)$$

$$z_1 = \beta_5 \cdot z_2 \quad (28)$$

$$z_2 = \beta_6 \cdot z_1 \quad (29)$$

where:  $x_1, y_1, z_1$  and  $x_2, y_2, z_2$  - the coordinates of the point **A** in the reference systems  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , respectively;

$t_1$  and  $t_2$  - the time value in the reference systems  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , respectively;

$\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$  and  $\beta_6$  - the transition coefficients;

$V_1$  - speed of system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  relative to the system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ .

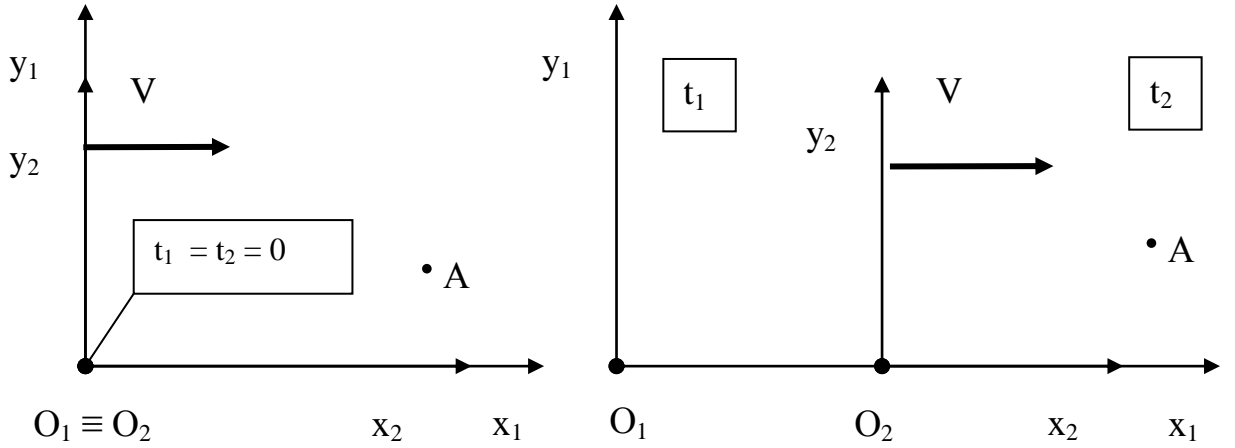


Fig. 1

Using the principle of relativity and the symmetry of the space and the time provides:

$$V_1 = -V_2 = V \quad (30)$$

$$\beta_1 = \beta_2 = \beta \quad (31)$$

$$\beta_3 = \beta_4 = 1 \quad (32)$$

$$\beta_5 = \beta_6 = 1 \quad (33)$$

This system of equations (24)-(29) are simplified and will take the form:

$$x_1 = \beta \cdot [x_2 + (V \cdot t_2)] \quad (34)$$

$$x_2 = \beta \cdot [x_1 - (V \cdot t_1)] \quad (35)$$

$$y_1 = y_2 \quad (36)$$

$$z_1 = z_2 \quad (37)$$

And the transition coefficient  $\beta$  is not dependent on the values of the coordinates  $x_1, y_1, z_1, x_2, y_2, z_2$  and the times  $t_1$  and  $t_2$ , and presumably could be a function of the speed  $V$  of the reference systems  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  about each other.

Of formulas (34) and (35) can be recorded the values of the times  $t_1$  and  $t_2$ :

$$t_1 = \frac{(\beta^2 - 1) \cdot x_2}{\beta \cdot V} + (\beta \cdot t_2) \quad (38)$$

$$t_2 = \frac{(1 - \beta^2) \cdot x_1}{\beta \cdot V} + (\beta \cdot t_1) \quad (39)$$

We can state the following for transition coefficient  $\beta$  in formulas (34) and (35):

- based on the principle of the relativity and the symmetry of space and the time, the transition coefficient  $\beta$  can only be a real value;
- the transition coefficient  $\beta$  will equal 1 with  $V = 0$  (the boundary condition);
- the transition coefficient  $\beta$  will equal 1 if it is not dependent on the speed  $V$ ;
- if the direction of the axis of the Cartesian coordinate of systems  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  is taken, the transition coefficient  $\beta$  will be more than 0. As well, negative values of the transition coefficient  $\beta$  transition will apply with a different direction of the axes  $\mathbf{O}_1\mathbf{x}_1$  and  $\mathbf{O}_2\mathbf{x}_2$ ;
- while the meaning of the transition coefficient  $\beta > 1$ , the linear dimension of the body moving on the inertial reference system, decreases in the direction of motion and time, moving on the same inertial reference system, slows;
- while the meaning of the transition coefficient  $0 < \beta < 1$  the linear dimension of the body, moving on the inertial reference system, increases in the direction of the movement and time, moving on the inertial reference system, accelerates;
- the principle of relativity and the symmetry of space and time determines that in the case of the application of the transition coefficient  $\beta$  on the values of the speed  $V$ , the transition coefficient  $\beta$  value unequivocally depends on the value of the speed  $V$  (that is, one specific value of the speed  $V$  applies to only one specific value of the transition coefficient  $\beta$ ).

Formulas (24)-(29) unequivocally define the accord between the coordinates  $x_1$ ,  $y_1$  and  $z_1$  of point  $A$  and time  $t_1$  in the fixed system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and the coordinates  $x_2$ ,  $y_2$  and  $z_2$  of the same point  $A$  and the time  $t_2$  in the moving system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ .



Using formulas (24)-(39), there may be obtained an unequivocal accord between the projection  $\mathbf{v}_{x2}$ ,  $\mathbf{v}_{y2}$  and  $\mathbf{v}_{z2}$  of the speed of point **A** in the moving system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  on the axis of the Cartesian coordinates and the similar projection  $\mathbf{v}_{x1}$ ,  $\mathbf{v}_{y1}$  and  $\mathbf{v}_{z1}$  of the speed of this point **A** in the fixed system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$ :

$$v_{x1} = \frac{v_{x2} + V}{\frac{(\beta^2 - 1) \cdot v_{x2}}{\beta^2 \cdot V} + 1} \quad (40)$$

$$v_{x2} = \frac{v_{x1} - V}{\frac{(1 - \beta^2) \cdot v_{x1}}{\beta^2 \cdot V} + 1} \quad (41)$$

$$v_{y1} = \frac{v_{y2}}{\frac{(\beta^2 - 1) \cdot v_{x2}}{\beta \cdot V} + \beta} \quad (42)$$

$$v_{y2} = \frac{v_{y1}}{\frac{(1 - \beta^2) \cdot v_{x1}}{\beta \cdot V} + \beta} \quad (43)$$

$$v_{z1} = \frac{v_{z2}}{\frac{(\beta^2 - 1) \cdot v_{x2}}{\beta \cdot V} + \beta} \quad (44)$$

$$v_{z2} = \frac{v_{z1}}{\frac{(1 - \beta^2) \cdot v_{x1}}{\beta \cdot V} + \beta} \quad (45)$$

Of formulas (38)-(45) can be obtained the unequivocal accord between the projection  $\mathbf{a}_{x2}$ ,  $\mathbf{a}_{y2}$  and  $\mathbf{a}_{z2}$  of the acceleration of point **A** in the moving system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  on the axis of the Cartesian coordinates and the similar projection  $\mathbf{a}_{x1}$ ,  $\mathbf{a}_{y1}$  and  $\mathbf{a}_{z1}$  of the acceleration of this point in the fixed system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  :

$$a_{x1} = \frac{a_{x2}}{\beta^3 \cdot \left[ \frac{(\beta^2 - 1) \cdot v_{x2}}{\beta^2 \cdot V} + 1 \right]^3} \quad (46)$$

$$a_{x2} = \frac{a_{x1}}{\beta^3 \cdot \left[ \frac{(1 - \beta^2) \cdot v_{x1}}{\beta^2 \cdot V} + 1 \right]^3} \quad (47)$$

$$a_{y1} = \frac{\left\{ a_{y2} \cdot \left[ \frac{(\beta^2 - 1) \cdot v_{x2}}{\beta \cdot V} + \beta \right] \right\} - \frac{(\beta^2 - 1) \cdot a_{x2} \cdot v_{y2}}{\beta \cdot V}}{\left[ \frac{(\beta^2 - 1) \cdot v_{x2}}{\beta \cdot V} + \beta \right]^3} \quad (48)$$

$$a_{y2} = \frac{\left\{ a_{y1} \cdot \left[ \frac{(1 - \beta^2) \cdot v_{x1}}{\beta \cdot V} + \beta \right] \right\} - \frac{(1 - \beta^2) \cdot a_{x1} \cdot v_{y1}}{\beta \cdot V}}{\left[ \frac{(1 - \beta^2) \cdot v_{x1}}{\beta \cdot V} + \beta \right]^3} \quad (49)$$

$$a_{z1} = \frac{\left\{ a_{z2} \cdot \left[ \frac{(\beta^2 - 1) \cdot v_{x2}}{\beta \cdot V} + \beta \right] \right\} - \frac{(\beta^2 - 1) \cdot a_{x2} \cdot v_{z2}}{\beta \cdot V}}{\left[ \frac{(\beta^2 - 1) \cdot v_{x2}}{\beta \cdot V} + \beta \right]^3} \quad (50)$$

$$a_{z2} = \frac{\left\{ a_{z1} \cdot \left[ \frac{(1 - \beta^2) \cdot v_{x1}}{\beta \cdot V} + \beta \right] \right\} - \frac{(1 - \beta^2) \cdot a_{x1} \cdot v_{z1}}{\beta \cdot V}}{\left[ \frac{(1 - \beta^2) \cdot v_{x1}}{\beta \cdot V} + \beta \right]^3} \quad (51)$$

## II.2. The definition of the special speed

Assume that there is the value  $V_{\text{xkp}}$  of the projection  $v_{x1}$  of the speed of point **A** in the fixed inertial reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$ , which would be consistent with the value of the projection  $v_{x2}$  of the speed of point **A** in the moving inertial reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  equal  $V_{\text{xkp}}$ . That is when:

$$v_{x1} = v_{x2} = V_{\text{xkp}} \quad (52)$$

Substituting value (52) in formulas (40) or (41), we get:

$$V_{\text{xkp}}^2 = \frac{\beta^2 \cdot V^2}{\beta^2 - 1} \quad (53)$$

Of formula (53), there should be an accord  $V_{\text{xkp}}$  with the speed  $V$  and the transition coefficient  $\beta$  for any possible values of speed  $V$ :

$$V_{\text{xkp}} = \pm \frac{\beta \cdot V}{\sqrt{\beta^2 - 1}} \quad (54)$$

In the event that the transition coefficient  $\beta$  is the value  $\beta \geq 1$ , we get that  $V_{\text{xkp}}$  will be a real value, which is written for further consideration as:

$$V_{\text{xkp}} = v_{\text{xkp1}} = \pm \frac{\beta \cdot V}{\sqrt{\beta^2 - 1}} \quad (55)$$

where:  $v_{\text{xkp1}}$  - the real value of having the speed dimension.

And if the transition coefficient  $\beta$  is the value  $0 < \beta \leq 1$ , we get that  $V_{\text{xkp}}$  will be an imaginary value (that suggests that the speed of the point in the fixed

reference system can never be equal to the speed of the same point in the moving inertial reference system with  $0 < \beta \leq 1$ ), which is written for further consideration as:

$$V_{\text{xkp}} = i \cdot v_{\text{xkp1}} = \pm \frac{i \cdot \beta \cdot V}{\sqrt{1 - \beta^2}} \quad (56)$$

where:  $v_{\text{xkp2}}$  - the real value of having the speed dimension, but  $i$  is equal:

$$i = \sqrt{-1} \quad (57)$$

From formula (53) there can be an accord for the transition coefficient  $\beta$  on the value of the speed  $V$  for any possible values of the speed  $V$ :

$$\beta^2 = \frac{1}{1 - \frac{V^2}{V_{\text{xkp}}^2}} \quad (58)$$

Then from formula (58), taking into account formula (55) for the transition coefficient  $\beta$ , and having values  $\beta \geq 1$  which is denoted as  $\beta_{>}$ , you can write:

$$\beta_{>}^2 = \frac{1}{1 - \frac{V^2}{V_{\text{xkp1}}^2}} \quad (59)$$

And from formula (58) taking into account formula (56) for the transition coefficient  $\beta$ , having values  $0 < \beta \leq 1$  and which is denoted as  $\beta_{<}$ , you can write:

$$\beta_{<}^2 = \frac{1}{1 + \frac{V^2}{V_{\text{xkp2}}^2}} \quad (60)$$

### II.3. The equation of accord for the transition coefficients

Consider three inertial reference systems: fixed  $O_1x_1y_1z_1$  and moving  $O_2x_2y_2z_2$  and  $O_3x_3y_3z_3$ , shown in Fig. 2 and from which:

- the axes of the Cartesian coordinate of the systems  $O_1x_1y_1z_1$ ,  $O_2x_2y_2z_2$  and  $O_3x_3y_3z_3$  are parallel and equally directed;
- system  $O_2x_2y_2z_2$  moving in system  $O_1x_1y_1z_1$  with constant speed  $V_2$  on the axis  $Ox_1$ ;
- system  $O_3x_3y_3z_3$  moving in the system  $O_1x_1y_1z_1$  with constant speed  $V_3$  on the axis  $Ox_1$ ;

- the starting time ( $t_1=0$ ,  $t_2=0$  and  $t_3=0$ ) in three systems is selected as when their coordinate centres  $\mathbf{O}_1$ ,  $\mathbf{O}_2$  and  $\mathbf{O}_3$  match.

Based on formula (41), you can determine the value of the speed  $\mathbf{V}_{23}$  of the motion of point  $\mathbf{O}_3$  on point  $\mathbf{O}_2$ :

$$V_{23} = \frac{V_3 - V_2}{\frac{(1 - \beta_2^2) \cdot V_3}{\beta_2^2 \cdot V_2} + 1} \quad (61)$$

and the value of speed  $\mathbf{V}_{32}$  of the motion of point  $\mathbf{O}_2$  on point  $\mathbf{O}_3$ :

$$V_{32} = \frac{V_2 - V_3}{\frac{(1 - \beta_3^2) \cdot V_2}{\beta_3^2 \cdot V_3} + 1} \quad (62)$$

where:  $\beta_2$  and  $\beta_3$  - the transition coefficients for the inertial reference systems, moving relative to the fixed reference system at speeds  $\mathbf{V}_2$  and  $\mathbf{V}_3$ , respectively.

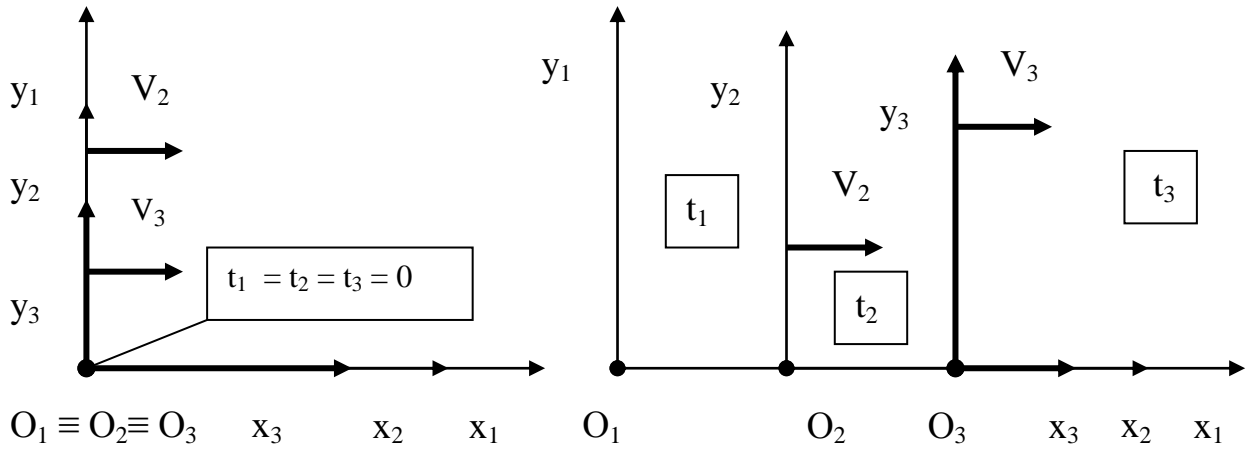


Fig. 2

Using the principle of relativity, point  $\mathbf{O}_3$  will be removed from point  $\mathbf{O}_2$  at speed equal in absolute value and opposite the speed of point  $\mathbf{O}_2$ , which is removed from point  $\mathbf{O}_3$ , that is:

$$V_{32} = - V_{23} \quad (63)$$

Substituting equation (63) in formulas (61) and (62), we get:

$$\frac{(1 - \beta_2^2) \cdot V_3}{\beta_2^2 \cdot V_2} + 1 = \frac{(1 - \beta_3^2) \cdot V_2}{\beta_3^2 \cdot V_3} + 1 \quad (64)$$

Hence the equation for the transition coefficients  $\beta_2$  and  $\beta_3$  are inscribed as

follows:

$$\beta_3^2 = \frac{\beta_2^2 \cdot V_2^2}{V_3^2 - (\beta_2^2 \cdot V_3^2) + (\beta_2^2 \cdot V_2^2)} \quad (65)$$

#### II.4. Getting accord for the transition coefficient $\beta$

From equation (64) may be obtained the formula:

$$\frac{\beta_2^2 - 1}{\beta_2^2 \cdot V_2^2} = \frac{\beta_3^2 - 1}{\beta_3^2 \cdot V_3^2} \quad (66)$$

Because the values of the transition coefficients  $\beta_2$  and  $\beta_3$  are independent of each other, and depend only on the values of the speeds  $V_2$  and  $V_3$ , respectively, and the values of the speeds  $V_2$  and  $V_3$  are raised arbitrarily (also not independent from each other), we can say that:

$$\frac{\beta_2^2 - 1}{\beta_2^2 \cdot V_2^2} = \frac{\beta_3^2 - 1}{\beta_3^2 \cdot V_3^2} = K = Const \quad (67)$$

that is in general terms:

$$\frac{\beta^2 - 1}{\beta^2 \cdot V^2} = K = Const \quad (68)$$

where:  $K$  - the constant value, is not dependent on the values of the speeds  $V$  ( $V_2$  and  $V_3$ ) and the values of the transition coefficient  $\beta$  ( $\beta_2$  and  $\beta_3$ ), and having the dimension of the inverse square of the speed.

As can be seen from formula (68), depending on the value of the constant  $K$  the transition coefficient  $\beta$  may have the following meanings:

- when  $K = 0$  the transition coefficient  $\beta$  will be equal 1,
- if the constant  $K$  is the real positive value, the transition coefficient  $\beta$  will be greater than or equal to 1, that is  $\beta \geq 1$ ,
- if the constant  $K$  is the real negative value, the transition coefficient  $\beta$  will be less or equal to 1, that is  $0 < \beta \leq 1$ .

But, as the constant  $K$  does not depend on the value of the speed  $V$  and the value of the transition coefficient  $\beta$ , then for any particular values of the speed  $V$  it turns out that the constant  $K$  can be both a positive value and a negative value. That

is, for all of the possible values of the speed  $\mathbf{V}$ , the value of the transition coefficient  $\beta$  can only be in the range of  $\beta \geq 1$  or only in the range  $0 < \beta \leq 1$ .

In short,  $\beta \geq 1$  and  $0 < \beta \leq 1$  are the two mutually exclusive ranges of the transition coefficient  $\beta$ . That is, all values of the transition coefficient  $\beta$  depending on speed  $\mathbf{V}$  are only in the range of  $\beta \geq 1$  or in the range  $0 < \beta \leq 1$ .

The main task is in the choice between these two ranges, which will really depend on the value of the transition coefficient  $\beta$ , depending on the value of the speed  $\mathbf{V}$  (if  $\beta$  depends on  $\mathbf{V}$ ).

The formula for the transition coefficient  $\beta$  may be obtained from equation (68):

$$\beta^2 = \frac{1}{1 - (K \cdot V^2)} \quad (69)$$

If you go back to formula (58):

$$\beta^2 = \frac{1}{1 - \frac{V^2}{V_{\text{кр}}^2}} \quad (58)$$

and compare it with the formula (69), it may be noted that:

$$K = \frac{1}{V_{\text{кр}}^2} \quad (70)$$

that is,  $V_{\text{кр}}^2$  will be the constant value, independent of the values of the speed  $\mathbf{V}$  and the transition coefficient  $\beta$ .

Based on the formulas (69) and (70), you can say, where the transition coefficient  $\beta$  is not equal to 1, there should be such that the speed  $V_{\text{кр}}$  (actual or perceived) of the motion of the point would be invariant in all directions and in all inertial reference systems.

Based on formula (69), in the formulas for the transition coefficient  $\beta$ :

- if  $\beta \geq 1$ :

$$\beta_{\geq}^2 = \frac{1}{1 - \frac{V^2}{V_{\text{кр}1}^2}} \quad (59)$$

- if  $0 < \beta \leq 1$ :

$$\beta_{<}^2 = \frac{1}{1 + \frac{V^2}{v_{\text{xkp2}}^2}} \quad (60)$$

the values  $v_{\text{xkp1}}$  and  $v_{\text{xkp2}}$  will fluctuate and are not dependent on the speed  $V$  and the transition coefficient  $\beta$ , that is:

$$v_{\text{xkp1}} = \text{Const} \quad (71)$$

$$v_{\text{xkp2}} = \text{Const} \quad (72)$$

The boundary condition (with the speed  $V$  equal to 0, the transition coefficient  $\beta$  equals 1) states that in the pursuit of the speed  $V$  to 0, the transition coefficient  $\beta$  moves towards 1, and this, according to the formulas (59) and (60), allows that:

$$v_{\text{xkp1}} \neq 0 \quad (73)$$

$$v_{\text{xkp2}} \neq 0 \quad (74)$$

And if the transition coefficient  $\beta$  does not depend on the value of the speed  $V$  (that is, if the value of the transition coefficient  $\beta = \text{Const} = 1$ ), then:

$$v_{\text{xkp1}} = \infty \quad (75)$$

$$v_{\text{xkp2}} = \infty \quad (76)$$

## **II.5. The major kinematic equations of the theory of the special relativity "in general terms"**

Substituting formula (58) in equations (34), (35), (38)-(39), (40)-(45) and (46)-(51), we get the following system of the equations:

$$x_1 = \frac{x_2 + (V \cdot t_2)}{\sqrt{1 - \frac{V^2}{v_{\text{xkp}}^2}}} \quad (77)$$

$$x_2 = \frac{x_1 - (V \cdot t_1)}{\sqrt{1 - \frac{V^2}{v_{\text{xkp}}^2}}} \quad (78)$$

$$t_1 = \frac{t_2 + \frac{V \cdot x_2}{V_{\text{хкр}}^2}}{\sqrt{1 - \frac{V^2}{V_{\text{хкр}}^2}}} \quad (79)$$

$$t_2 = \frac{t_1 - \frac{V \cdot x_1}{V_{\text{хкр}}^2}}{\sqrt{1 - \frac{V^2}{V_{\text{хкр}}^2}}} \quad (80)$$

$$v_{x1} = \frac{v_{x2} + V}{1 + \frac{V \cdot v_{x2}}{V_{\text{хкр}}^2}} \quad (81)$$

$$v_{x2} = \frac{v_{x1} - V}{1 - \frac{V \cdot v_{x1}}{V_{\text{хкр}}^2}} \quad (82)$$

$$v_{y1} = \frac{v_{y2} \cdot \sqrt{1 - \frac{V^2}{V_{\text{хкр}}^2}}}{1 + \frac{V \cdot v_{x2}}{V_{\text{хкр}}^2}} \quad (83)$$

$$v_{y2} = \frac{v_{y1} \cdot \sqrt{1 - \frac{V^2}{V_{\text{хкр}}^2}}}{1 - \frac{V \cdot v_{x1}}{V_{\text{хкр}}^2}} \quad (84)$$

$$v_{z1} = \frac{v_{z2} \cdot \sqrt{1 - \frac{V^2}{V_{\text{хкр}}^2}}}{1 + \frac{V \cdot v_{x2}}{V_{\text{хкр}}^2}} \quad (85)$$

$$v_{z2} = \frac{v_{z1} \cdot \sqrt{1 - \frac{V^2}{V_{\text{хкр}}^2}}}{1 - \frac{V \cdot v_{x1}}{V_{\text{хкр}}^2}} \quad (86)$$



$$a_{x1} = \frac{a_{x2} \cdot \left( \sqrt{1 - \frac{V^2}{V_{\text{кр}}^2}} \right)^3}{\left( 1 + \frac{V \cdot v_{x2}}{V_{\text{кр}}^2} \right)^3} \quad (87)$$

$$a_{x2} = \frac{a_{x1} \cdot \left( \sqrt{1 - \frac{V^2}{V_{\text{кр}}^2}} \right)^3}{\left( 1 - \frac{V \cdot v_{x1}}{V_{\text{кр}}^2} \right)^3} \quad (88)$$

$$a_{y1} = \frac{\left\{ \left[ a_{y2} \cdot \left( 1 + \frac{V \cdot v_{x2}}{V_{\text{кр}}^2} \right) \right] - \frac{V \cdot a_{x2} \cdot v_{y2}}{V_{\text{кр}}^2} \right\} \cdot \left( 1 - \frac{V^2}{V_{\text{кр}}^2} \right)}{\left( 1 + \frac{V \cdot v_{x2}}{V_{\text{кр}}^2} \right)^3} \quad (89)$$

$$a_{y2} = \frac{\left\{ \left[ a_{y1} \cdot \left( 1 - \frac{V \cdot v_{x1}}{V_{\text{кр}}^2} \right) \right] + \frac{V \cdot a_{x1} \cdot v_{y1}}{V_{\text{кр}}^2} \right\} \cdot \left( 1 - \frac{V^2}{V_{\text{кр}}^2} \right)}{\left( 1 - \frac{V \cdot v_{x1}}{V_{\text{кр}}^2} \right)^3} \quad (90)$$

$$a_{z1} = \frac{\left\{ \left[ a_{z2} \cdot \left( 1 + \frac{V \cdot v_{x2}}{V_{\text{кр}}^2} \right) \right] - \frac{V \cdot a_{x2} \cdot v_{z2}}{V_{\text{кр}}^2} \right\} \cdot \left( 1 - \frac{V^2}{V_{\text{кр}}^2} \right)}{\left( 1 + \frac{V \cdot v_{x2}}{V_{\text{кр}}^2} \right)^3} \quad (91)$$

$$a_{z2} = \frac{\left\{ \left[ a_{z1} \cdot \left( 1 - \frac{V \cdot v_{x1}}{V_{\text{кр}}^2} \right) \right] + \frac{V \cdot a_{x1} \cdot v_{z1}}{V_{\text{кр}}^2} \right\} \cdot \left( 1 - \frac{V^2}{V_{\text{кр}}^2} \right)}{\left( 1 - \frac{V \cdot v_{x1}}{V_{\text{кр}}^2} \right)^3} \quad (92)$$

## II.6. The major kinematic equations when the transition coefficient $\beta \geq 1$

Substituting formula (59) in (34), (35), (38)-(39), (40)-(45) and (46)-(51), we get the following system of equations with transition coefficient  $\beta = \beta_>$ :

$$x_{1>} = \frac{x_{2>} + (V \cdot t_{2>})}{\sqrt{1 - \frac{V^2}{v_{\text{кр}1}^2}}} \quad (93)$$

$$x_{2>} = \frac{x_{1>} - (V \cdot t_{1>})}{\sqrt{1 - \frac{V^2}{v_{\text{кр}1}^2}}} \quad (94)$$

$$t_{1>} = \frac{t_{2>} + \frac{V \cdot x_{2>}}{v_{\text{кр}1}^2}}{\sqrt{1 - \frac{V^2}{v_{\text{кр}1}^2}}} \quad (95)$$

$$t_{2>} = \frac{t_{1>} - \frac{V \cdot x_{1>}}{v_{\text{кр}1}^2}}{\sqrt{1 - \frac{V^2}{v_{\text{кр}1}^2}}} \quad (96)$$

$$v_{x1>} = \frac{v_{x2>} + V}{1 + \frac{V \cdot v_{x2>}}{v_{\text{кр}1}^2}} \quad (97)$$

$$v_{x2>} = \frac{v_{x1>} - V}{1 - \frac{V \cdot v_{x1>}}{v_{\text{кр}1}^2}} \quad (98)$$

$$v_{y1>} = \frac{v_{y2>} \cdot \sqrt{1 - \frac{V^2}{v_{\text{кр}1}^2}}}{1 + \frac{V \cdot v_{x2>}}{v_{\text{кр}1}^2}} \quad (99)$$

$$v_{y2>} = \frac{v_{y1>} \cdot \sqrt{1 - \frac{V^2}{v_{\text{кр}1}^2}}}{1 - \frac{V \cdot v_{x1>}}{v_{\text{кр}1}^2}} \quad (100)$$

$$v_{z1>} = \frac{v_{z2>} \cdot \sqrt{1 - \frac{V^2}{v_{\text{хкр}1}^2}}}{1 + \frac{V \cdot v_{x2>}}{v_{\text{хкр}1}^2}} \quad (101)$$

$$v_{z2>} = \frac{v_{z1>} \cdot \sqrt{1 - \frac{V^2}{v_{\text{хкр}1}^2}}}{1 - \frac{V \cdot v_{x1>}}{v_{\text{хкр}1}^2}} \quad (102)$$

$$a_{x1>} = \frac{a_{x2>} \cdot \left( \sqrt{1 - \frac{V^2}{v_{\text{хкр}1}^2}} \right)^3}{\left( 1 + \frac{V \cdot v_{x2>}}{v_{\text{хкр}1}^2} \right)^3} \quad (103)$$

$$a_{x2>} = \frac{a_{x1>} \cdot \left( \sqrt{1 - \frac{V^2}{v_{\text{хкр}1}^2}} \right)^3}{\left( 1 - \frac{V \cdot v_{x1>}}{v_{\text{хкр}1}^2} \right)^3} \quad (104)$$

$$a_{y1>} = \frac{\left\{ \left[ a_{y2>} \cdot \left( 1 + \frac{V \cdot v_{x2>}}{v_{\text{хкр}1}^2} \right) \right] - \frac{V \cdot a_{x2>} \cdot v_{y2>}}{v_{\text{хкр}1}^2} \right\} \cdot \left( 1 - \frac{V^2}{v_{\text{хкр}1}^2} \right)}{\left( 1 + \frac{V \cdot v_{x2>}}{v_{\text{хкр}1}^2} \right)^3} \quad (105)$$

$$a_{y2>} = \frac{\left\{ \left[ a_{y1>} \cdot \left( 1 - \frac{V \cdot v_{x1>}}{v_{\text{хкр}1}^2} \right) \right] + \frac{V \cdot a_{x1>} \cdot v_{y1>}}{v_{\text{хкр}1}^2} \right\} \cdot \left( 1 - \frac{V^2}{v_{\text{хкр}1}^2} \right)}{\left( 1 - \frac{V \cdot v_{x1>}}{v_{\text{хкр}1}^2} \right)^3} \quad (106)$$

$$a_{z1>} = \frac{\left\{ \left[ a_{z2>} \cdot \left( 1 + \frac{V \cdot v_{x2>}}{v_{xkp1}^2} \right) \right] - \frac{V \cdot a_{x2>} \cdot v_{z2>}}{v_{xkp1}^2} \right\} \cdot \left( 1 - \frac{V^2}{v_{xkp1}^2} \right)}{\left( 1 + \frac{V \cdot v_{x2>}}{v_{xkp1}^2} \right)^3} \quad (107)$$

$$a_{z2>} = \frac{\left\{ \left[ a_{z1>} \cdot \left( 1 - \frac{V \cdot v_{x1>}}{v_{xkp1}^2} \right) \right] + \frac{V \cdot a_{x1>} \cdot v_{z1>}}{v_{xkp1}^2} \right\} \cdot \left( 1 - \frac{V^2}{v_{xkp1}^2} \right)}{\left( 1 - \frac{V \cdot v_{x1>}}{v_{xkp1}^2} \right)^3} \quad (108)$$

## II.7. The major kinematic equations when the transition coefficient is $0 < \beta < 1$

Substituting formula (60) in equations (34), (35), (38)-(39)), (40)-(45) and (46)-(51), we get the following system of the equations when the transition coefficient is  $\beta = \beta_<$ :

$$x_{1<} = \frac{x_{2<} + (V \cdot t_{2<})}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \quad (109)$$

$$x_{2<} = \frac{x_{1<} - (V \cdot t_{1<})}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \quad (110)$$

$$t_{1<} = \frac{t_{2<} - \frac{V \cdot x_{2<}}{v_{xkp2}^2}}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \quad (111)$$

$$t_{2<} = \frac{t_{1<} + \frac{V \cdot x_{1<}}{v_{xkp2}^2}}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \quad (112)$$

$$v_{x1<} = \frac{v_{x2<} + V}{1 - \frac{V \cdot v_{x2<}}{v_{xkp2}^2}} \quad (113)$$

$$v_{x2<} = \frac{v_{x1<} - V}{1 + \frac{V \cdot v_{x1<}}{v_{\text{хкр}2}^2}} \quad (114)$$

$$v_{y1<} = \frac{v_{y2<} \cdot \sqrt{1 + \frac{V^2}{v_{\text{хкр}2}^2}}}{1 - \frac{V \cdot v_{x2<}}{v_{\text{хкр}2}^2}} \quad (115)$$

$$v_{y2<} = \frac{v_{y1<} \cdot \sqrt{1 + \frac{V^2}{v_{\text{хкр}2}^2}}}{1 + \frac{V \cdot v_{x1<}}{v_{\text{хкр}2}^2}} \quad (116)$$

$$v_{z1<} = \frac{v_{z2<} \cdot \sqrt{1 + \frac{V^2}{v_{\text{хкр}2}^2}}}{1 - \frac{V \cdot v_{x2<}}{v_{\text{хкр}2}^2}} \quad (117)$$

$$v_{z2<} = \frac{v_{z1<} \cdot \sqrt{1 + \frac{V^2}{v_{\text{хкр}2}^2}}}{1 + \frac{V \cdot v_{x1<}}{v_{\text{хкр}2}^2}} \quad (118)$$

$$a_{x1<} = \frac{a_{x2<} \cdot \left( \sqrt{1 + \frac{V^2}{v_{\text{хкр}1}^2}} \right)^3}{\left( 1 - \frac{V \cdot v_{x2<}}{v_{\text{хкр}1}^2} \right)^3} \quad (119)$$

$$a_{x2<} = \frac{a_{x1<} \cdot \left( \sqrt{1 + \frac{V^2}{v_{xkp2}^2}} \right)^3}{\left( 1 + \frac{V \cdot v_{x1<}}{v_{xkp2}^2} \right)^3} \quad (120)$$

$$a_{y1<} = \frac{\left\{ \left[ a_{y2<} \cdot \left( 1 - \frac{V \cdot v_{x2<}}{v_{xkp2}^2} \right) \right] + \frac{V \cdot a_{x2<} \cdot v_{y2<}}{v_{xkp2}^2} \right\} \cdot \left( 1 + \frac{V^2}{v_{xkp2}^2} \right)}{\left( 1 - \frac{V \cdot v_{x2<}}{v_{xkp2}^2} \right)^3} \quad (121)$$

$$a_{y2<} = \frac{\left\{ \left[ a_{y1<} \cdot \left( 1 + \frac{V \cdot v_{x1<}}{v_{xkp2}^2} \right) \right] - \frac{V \cdot a_{x1<} \cdot v_{y1<}}{v_{xkp2}^2} \right\} \cdot \left( 1 + \frac{V^2}{v_{xkp2}^2} \right)}{\left( 1 + \frac{V \cdot v_{x1<}}{v_{xkp2}^2} \right)^3} \quad (122)$$

$$a_{z1<} = \frac{\left\{ \left[ a_{z2<} \cdot \left( 1 - \frac{V \cdot v_{x2<}}{v_{xkp2}^2} \right) \right] + \frac{V \cdot a_{x2<} \cdot v_{z2<}}{v_{xkp2}^2} \right\} \cdot \left( 1 + \frac{V^2}{v_{xkp2}^2} \right)}{\left( 1 - \frac{V \cdot v_{x2<}}{v_{xkp2}^2} \right)^3} \quad (123)$$

$$a_{z2<} = \frac{\left\{ \left[ a_{z1<} \cdot \left( 1 + \frac{V \cdot v_{x1<}}{v_{xkp2}^2} \right) \right] - \frac{V \cdot a_{x1<} \cdot v_{z1<}}{v_{xkp2}^2} \right\} \cdot \left( 1 + \frac{V^2}{v_{xkp2}^2} \right)}{\left( 1 + \frac{V \cdot v_{x1<}}{v_{xkp2}^2} \right)^3} \quad (124)$$

Unfortunately, the connection of kinematic equations to determine the meaning of the constant value  $\mathbf{V}_{xkp}$  ( $\mathbf{v}_{xkp1}$  or  $\mathbf{v}_{xkp2}$ ) is not possible. Therefore, we have to turn to dynamics.

### III. The dynamics

In order to the establish the moving body's mass dependence on speed based on the one hand - on the principle of the relativity, claiming, that the physical laws are invariant with the respect to the choice of the inertial reference systems. That is, the equations expressing these laws have the same form in all inertial reference systems.

On the other hand - try to rely on the restrictive conditions of space and time, which are set in the special theory of relativity.

These conditions are the homogeneity and the isotropy of space and the homogeneity of time. That is, the symmetry of space and time.

According to Emmy Noether's theorem, the symmetry of actions match the law of conservation of these actions.

Emmy Noether's theorem allows that for:

- the law of the conservation of mechanical energy associated with the properties of the symmetry of time - the homogeneousness of time (this feature of time is reflected in the fact that the laws of the movement of a closed system do not depend on the choice of the starting time);

- the law of the conservation of the momentum associated with the properties of the symmetry of space - the homogeneousness of space (this feature of the space is reflected in the fact that the physical properties of a closed system and its laws of motion do not depend on the choice of the provisions of the center coordinates of the inertial reference system. That is, not modified in parallel transfer of a closed system as a whole in space);

- the law of conservation of angular momentum associated with the property of the symmetry of space - the isotropy of space (this feature of space was reflected in the fact that the physical properties and the laws of motion of a closed system do not depend on the choice of directions of the coordinate axes of the inertial reference system. That is, not modified while turning the closed system as the whole at any angle in space).

### **III.1. The system of equations to determine the mass of a moving body based on speed**

In order to determine the mass of a moving body, use:

- the law of conservation of momentum: **the momentum of a closed mechanical system of bodies (in which outside forces do not exist) for any time, is a constant value;**

- the law of the conservation of mechanical energy: **the mechanical energy of the conservative mechanical system of the bodies (in which all internal forces are potential and all external forces are potential and stationary) for any time is a constant value.** This kind of closed mechanical system takes the form of: **the mechanical energy of a closed mechanical system does not change over time if all internal forces operating in the system are potential,** more precisely - the particular case when the bodies forming the closed mechanical system, do not change the potential energy (including where the bodies forming the closed mechanical system have totally elastic interactions): **the kinetic energy of a closed mechanical system of bodies for any time is a constant value.**

Assume, that the determination of mass of a moving body based on its speed does not change with a change in the potential energy of the body.

Suppose, that mass  $M(V)$  of a material point, moving at the speed  $V$ , is equal to:

$$M(V) = M_0 \cdot f(V) \quad (125)$$

where:  $M_0$  - the mass  $M(V)$  of the material point in a state of rest;

$f(V)$  - is the function presumably depending on the speed  $V$ .

Based on formula (125), the momentum  $P(V)$  of the material point, moving at speed  $V$ , is equal to:

$$P(V) = M_0 \cdot f(V) \cdot V \quad (126)$$

Formula (126) lets you write the following equation for the kinetic energy  $E_k(V)$  of the material point, moving at speed  $V$ :

$$E_k(V) = M_0 \cdot \int_0^V \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \quad (127)$$

where:  $f'(V)$  is the derivative of the function  $f(V)$ .

Let's try to establish the mass of the moving body (the function  $f(V)$ ) on its speed, having examined the interaction (or rather the result of the interaction) of the bodies (the material points), forming the closed mechanical system and moving only linearly in the space.



With the aim of writing the systems of equations for the determining the value of the function  $f(\mathbf{V})$ , look at two simple examples.

### III.1.1. Example № 1

Assume that there are two inertial reference systems similar to the reference system shown in Fig. 1, fixed  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and moving  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , at speed  $\mathbf{V}$  parallel to the axis  $\mathbf{O}_1\mathbf{x}_1$  relative system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$ .

Assume that there is a closed mechanical system of bodies, comprised of body 1 and the body 2, as shown in Fig.3, with the masses in a state of rest, equaling  $\mathbf{M}_{01}$  and  $\mathbf{M}_{02}$ , respectively.

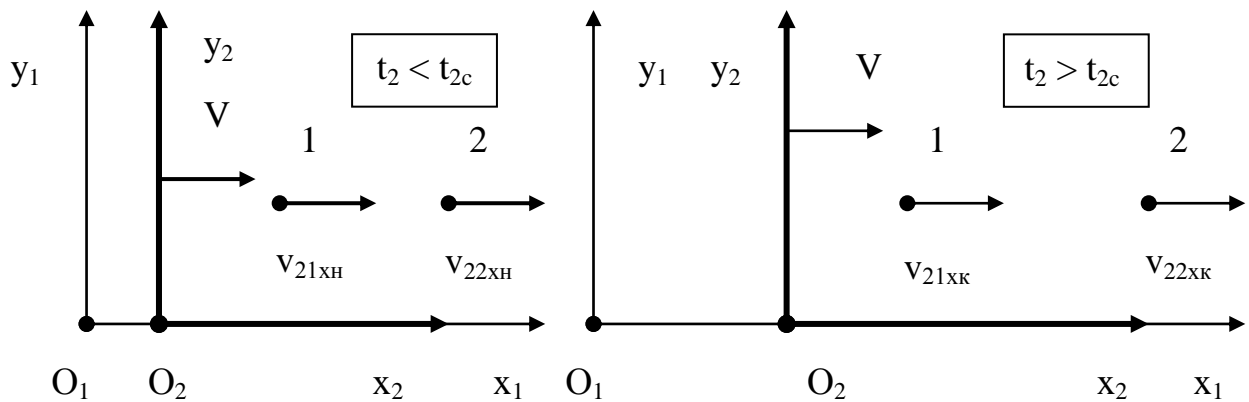


Fig. 3

Until time  $t_{2c}$  in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  body 1 and body 2 are moving parallel to the  $\mathbf{O}_2\mathbf{x}_2$  axis on one line with constant speeds  $\mathbf{v}_{21xH}$  and  $\mathbf{v}_{22xH}$  respectively.

At some point in time  $t_{2c}$  between bodies 1 and 2 there is a central, direct, absolutely elastic collision.

After the collision at time  $t_{2c}$  bodies 1 and 2 begin moving parallel to the  $\mathbf{O}_2\mathbf{x}_2$  axis on one line with constant speeds  $\mathbf{v}_{21xK}$  and  $\mathbf{v}_{22xK}$ , respectively.

Given that between bodies 1 and 2 there has been a central direct collision, and treating them as material points, write the law of the conservation of momentum for the closed mechanical system of bodies 1 and 2 for moments of time, less than and greater than  $t_{2c}$ , in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ :

$$\begin{aligned}
& [M_{01} \cdot f(V = v_{21xH}) \cdot v_{21xH}] + [M_{02} \cdot f(V = v_{22xH}) \cdot v_{22xH}] = \\
& = [M_{01} \cdot f(V = v_{21xK}) \cdot v_{21xK}] + [M_{02} \cdot f(V = v_{22xK}) \cdot v_{22xK}] \quad (128)
\end{aligned}$$

Assuming that the collision of bodies 1 and 2 was absolutely elastic, the law of the conservation of mechanical energy to the closed mechanical system of bodies 1 and 2 can be written for the moments of time,  $t$  less than and greater than  $t_{2c}$ , in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , assuming that the values of the potential energy of the bodies remain unchanged before and after the collision:

$$\begin{aligned}
& \langle M_{01} \cdot \int_0^{v_{21xH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\
& + \langle M_{02} \cdot \int_0^{v_{22xH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle = \\
& = \langle M_{01} \cdot \int_0^{v_{21xK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\
& + \langle M_{02} \cdot \int_0^{v_{22xK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle \quad (129)
\end{aligned}$$

All that has been said earlier about the movement of bodies 1 and 2 in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  can be said about the movement of bodies 1 and 2 in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$ , except that:

- the collision between bodies 1 and 2 happens at time  $t_{1c}$  relative to time  $t_{2c}$  in the system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ ,

- body 1 is, respectively, before and after the collision at speeds  $\mathbf{v}_{11xH}$  and  $\mathbf{v}_{11xK}$  relative to the speeds  $\mathbf{v}_{21xH}$  and  $\mathbf{v}_{21xK}$ ,

- body 2 is, respectively, before and after the collision at speeds  $\mathbf{v}_{12xH}$  and  $\mathbf{v}_{12xK}$  relative to speeds  $\mathbf{v}_{22xH}$  and  $\mathbf{v}_{22xK}$ .

Similarly, formulas (128) and (129) can be written as the law of conservation of momentum and the law of conservation of mechanical energy for the closed mechanical system of bodies 1 and 2 for the moments of time, smaller and greater than  $t_{1c}$ , in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  also assuming that the potential energy of bodies 1 and 2 remain unchanged before and after the collision:

$$\begin{aligned}
& [M_{01} \cdot f(V = v_{11xH}) \cdot v_{11xH}] + [M_{02} \cdot f(V = v_{12xH}) \cdot v_{12xH}] = \\
& = [M_{01} \cdot f(V = v_{11xK}) \cdot v_{11xK}] + [M_{02} \cdot f(V = v_{12xK}) \cdot v_{12xK}] \quad (130)
\end{aligned}$$

$$\begin{aligned}
& \langle M_{01} \cdot \int_0^{v_{11xH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\
& + \langle M_{02} \cdot \int_0^{v_{12xH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle = \\
& = \langle M_{01} \cdot \int_0^{v_{11xK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\
& + \langle M_{02} \cdot \int_0^{v_{12xK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle \quad (131)
\end{aligned}$$

### III.1.2. The example № 2

Example № 2 is similar to example № 1, and differs only, that in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , body 1 and body 2 are moving non-parallel to axis  $\mathbf{O}_2\mathbf{x}_2$ , and are moving parallel to axis  $\mathbf{O}_2\mathbf{y}_2$ , as shown in Fig. 4.

Until some time  $t_{2c}$  in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , body 1 and body 2 are moving parallel to axis  $\mathbf{O}_2\mathbf{y}_2$  and on one line with the constant speeds  $v_{21yH}$  and the largest,  $v_{22yH}$ , respectively.

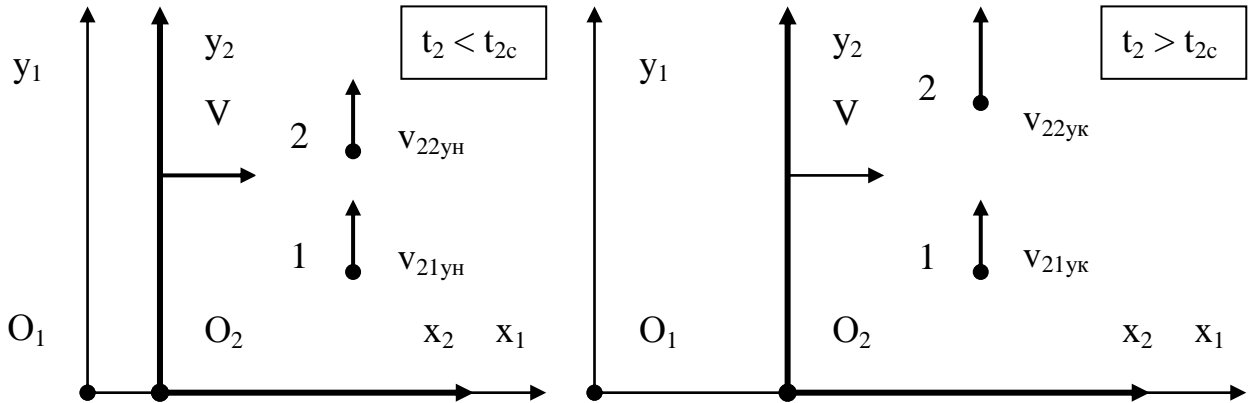


Fig.4

After the collision at greater time of  $t_{2c}$  bodies 1 and 2 begin moving parallel to the  $\mathbf{O}_2\mathbf{y}_2$  axis with constant speeds  $\mathbf{v}_{21y_K}$  and the larger,  $\mathbf{v}_{22y_K}$  respectively.

Then you can write the law of conservation of momentum and the law of conservation of mechanical energy in the closed mechanical system of bodies 1 and 2 for the moments of time, smaller and larger than  $t_{2c}$  in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , assuming that the potential energy of the bodies remains unchanged before and after the collision:

$$\begin{aligned} & [M_{01} \cdot f(V = v_{21y_H}) \cdot v_{21y_H}] + [M_{02} \cdot f(V = v_{22y_H}) \cdot v_{22y_H}] = \\ & = [M_{01} \cdot f(V = v_{21y_K}) \cdot v_{21y_K}] + [M_{02} \cdot f(V = v_{22y_K}) \cdot v_{22y_K}] \quad (132) \end{aligned}$$

$$\begin{aligned} & \langle M_{01} \cdot \int_0^{v_{21y_H}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\ & + \langle M_{02} \cdot \int_0^{v_{22y_H}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle = \\ & = \langle M_{01} \cdot \int_0^{v_{21y_K}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\ & + \langle M_{02} \cdot \int_0^{v_{22y_K}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle \quad (133) \end{aligned}$$

Given that the movement of bodies 1 and 2 in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  unlike their movement in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  can say that:

- the collision between bodies 1 and 2 happens at time  $t_{1c}$  relative to time  $t_{2c}$  in the system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ ,

- body 1 is, respectively, before and after the collision at the projections of speed  $\mathbf{v}_{11x_H} = \mathbf{V}$ ,  $\mathbf{v}_{11y_H}$ ,  $\mathbf{v}_{11x_K} = \mathbf{V}$  and  $\mathbf{v}_{11y_K}$  on axis coordinate  $\mathbf{O}_1\mathbf{x}_1$  and  $\mathbf{O}_1\mathbf{y}_1$  relative to the speeds  $\mathbf{v}_{21x_H}$  and  $\mathbf{v}_{21x_K}$ ,

- body 2 is, respectively, before and after the collision at the projections of speed  $\mathbf{v}_{12x_H} = \mathbf{V}$ ,  $\mathbf{v}_{12y_H}$ ,  $\mathbf{v}_{12x_K} = \mathbf{V}$  and  $\mathbf{v}_{12y_K}$  on axis coordinate  $\mathbf{O}_1\mathbf{x}_1$  and  $\mathbf{O}_1\mathbf{y}_1$  relative to the speeds  $\mathbf{v}_{22x_H}$  and  $\mathbf{v}_{22x_K}$ .

Similarly, you can write the law of conservation of momentum (two equations for the projections of momentum on axis  $\mathbf{O}_1\mathbf{x}_1$  and  $\mathbf{O}_1\mathbf{y}_1$ ) and the law of conservation of mechanical energy for the closed mechanical system of bodies 1 and 2 for the moments of time, smaller and larger than  $t_{1c}$ , in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$ , assuming that the potential energy of bodies 1 and 2 remain unchanged before and after the collision:

$$\begin{aligned} & \left[ M_{01} \cdot f\left(V = \sqrt{v_{11yH}^2 + V^2}\right) \cdot V \right] + \left[ M_{02} \cdot f\left(V = \sqrt{v_{12yH}^2 + V^2}\right) \cdot V \right] = \\ & = \left[ M_{01} \cdot f\left(V = \sqrt{v_{11yK}^2 + V^2}\right) \cdot V \right] + \left[ M_{02} \cdot f\left(V = \sqrt{v_{12yK}^2 + V^2}\right) \cdot V \right] \quad (134) \end{aligned}$$

$$\begin{aligned} & \left[ M_{01} \cdot f\left(V = \sqrt{v_{11yH}^2 + V^2}\right) \cdot v_{11yH} \right] + \left[ M_{02} \cdot f\left(V = \sqrt{v_{12yH}^2 + V^2}\right) \cdot v_{12yH} \right] = \\ & = \left[ M_{01} \cdot f\left(V = \sqrt{v_{11yK}^2 + V^2}\right) \cdot v_{11yK} \right] + \\ & \quad + \left[ M_{02} \cdot f\left(V = \sqrt{v_{12yK}^2 + V^2}\right) \cdot v_{12yK} \right] \quad (135) \end{aligned}$$

$$\begin{aligned} & \langle M_{01} \cdot \int_0^{\sqrt{v_{11yH}^2 + V^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\ & + \langle M_{02} \cdot \int_0^{\sqrt{v_{12yH}^2 + V^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle = \\ & = \langle M_{01} \cdot \int_0^{\sqrt{v_{11yK}^2 + V^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\ & + \langle M_{02} \cdot \int_0^{\sqrt{v_{12yK}^2 + V^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle \quad (136) \end{aligned}$$

### III.1.3. The formula for the correspondence of the speed and mass of the moving body

In order to calculate the mass of the moving body, consider the following system of the equations:

$$\begin{aligned} & [M_{01} \cdot f(V = v_{21xH}) \cdot v_{21xH}] + [M_{02} \cdot f(V = v_{22xH}) \cdot v_{22xH}] = \\ & = [M_{01} \cdot f(V = v_{21xK}) \cdot v_{21xK}] + [M_{02} \cdot f(V = v_{22xK}) \cdot v_{22xK}] \end{aligned} \quad (128)$$

$$\begin{aligned} & \langle M_{01} \cdot \int_0^{v_{21xH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\ & + \langle M_{02} \cdot \int_0^{v_{22xH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle = \\ & = \langle M_{01} \cdot \int_0^{v_{21xK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\ & + \langle M_{02} \cdot \int_0^{v_{22xK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle \end{aligned} \quad (129)$$

$$\begin{aligned} & [M_{01} \cdot f(V = v_{11xH}) \cdot v_{11xH}] + [M_{02} \cdot f(V = v_{12xH}) \cdot v_{12xH}] = \\ & = [M_{01} \cdot f(V = v_{11xK}) \cdot v_{11xK}] + [M_{02} \cdot f(V = v_{12xK}) \cdot v_{12xK}] \end{aligned} \quad (130)$$

$$\begin{aligned} & \langle M_{01} \cdot \int_0^{v_{11xH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\ & + \langle M_{02} \cdot \int_0^{v_{12xH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle = \\ & = \langle M_{01} \cdot \int_0^{v_{11xK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\ & + \langle M_{02} \cdot \int_0^{v_{12xK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle \end{aligned} \quad (131)$$

$$\begin{aligned} & [M_{01} \cdot f(V = v_{21yH}) \cdot v_{21yH}] + [M_{02} \cdot f(V = v_{22yH}) \cdot v_{22yH}] = \\ & = [M_{01} \cdot f(V = v_{21yK}) \cdot v_{21yK}] + [M_{02} \cdot f(V = v_{22yK}) \cdot v_{22yK}] \end{aligned} \quad (132)$$

$$\begin{aligned}
& \langle M_{01} \cdot \int_0^{v_{21yH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\
& + \langle M_{02} \cdot \int_0^{v_{22yH}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle = \\
& = \langle M_{01} \cdot \int_0^{v_{21yK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\
& + \langle M_{02} \cdot \int_0^{v_{22yK}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle \quad (133)
\end{aligned}$$

$$\begin{aligned}
& \left[ M_{01} \cdot f\left(V = \sqrt{v_{11yH}^2 + V^2}\right) \cdot V \right] + \left[ M_{02} \cdot f\left(V = \sqrt{v_{12yH}^2 + V^2}\right) \cdot V \right] = \\
& = \left[ M_{01} \cdot f\left(V = \sqrt{v_{11yK}^2 + V^2}\right) \cdot V \right] + \left[ M_{02} \cdot f\left(V = \sqrt{v_{12yK}^2 + V^2}\right) \cdot V \right] \quad (134)
\end{aligned}$$

$$\begin{aligned}
& \left[ M_{01} \cdot f\left(V = \sqrt{v_{11yH}^2 + V^2}\right) \cdot v_{11yH} \right] + \left[ M_{02} \cdot f\left(V = \sqrt{v_{12yH}^2 + V^2}\right) \cdot v_{12yH} \right] = \\
& = \left[ M_{01} \cdot f\left(V = \sqrt{v_{11yK}^2 + V^2}\right) \cdot v_{11yK} \right] + \\
& + \left[ M_{02} \cdot f\left(V = \sqrt{v_{12yK}^2 + V^2}\right) \cdot v_{12yK} \right] \quad (135)
\end{aligned}$$

$$\begin{aligned}
& \langle M_{01} \cdot \int_0^{\sqrt{v_{11yH}^2 + V^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\
& + \langle M_{02} \cdot \int_0^{\sqrt{v_{12yH}^2 + V^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle = \\
& = \langle M_{01} \cdot \int_0^{\sqrt{v_{11yK}^2 + V^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle + \\
& + \langle M_{02} \cdot \int_0^{\sqrt{v_{12yK}^2 + V^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \rangle \quad (136)
\end{aligned}$$

In the system of equations must also be added the equation of the accord between the speed projection of bodies 1 and 2 in the moving  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  and fixed  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  systems, recorded in formulas (81) and (83):

$$v_{11xH} = \frac{v_{21xH} + V}{1 + \frac{V \cdot v_{21xH}}{V_{xkp}^2}} \quad (137)$$

$$v_{12xH} = \frac{v_{22xH} + V}{1 + \frac{V \cdot v_{22xH}}{V_{xkp}^2}} \quad (138)$$

$$v_{11xK} = \frac{v_{21xK} + V}{1 + \frac{V \cdot v_{21xK}}{V_{xkp}^2}} \quad (139)$$

$$v_{12xK} = \frac{v_{22xK} + V}{1 + \frac{V \cdot v_{22xK}}{V_{xkp}^2}} \quad (140)$$

$$v_{11yH} = v_{21yH} \cdot \sqrt{1 - \frac{V^2}{V_{xkp}^2}} \quad (141)$$

$$v_{11yK} = v_{21yK} \cdot \sqrt{1 - \frac{V^2}{V_{xkp}^2}} \quad (142)$$

$$v_{12yH} = v_{22yH} \cdot \sqrt{1 - \frac{V^2}{V_{xkp}^2}} \quad (143)$$

$$v_{12yK} = v_{22yK} \cdot \sqrt{1 - \frac{V^2}{V_{xkp}^2}} \quad (144)$$

And so there are 17 equations, 12 unknown values and one unknown function.

The only function  $f(V)$ , that will meet the all requirements of 17 equations,

is:

$$f(V) = \frac{1}{\sqrt{1 - \frac{V^2}{V_{xkp}^2}}} \quad (145)$$



Then, taking into account equations (125)-(127) you can write the dependencies of the moving mass  $\mathbf{M}(\mathbf{V})$ , momentum  $\mathbf{P}(\mathbf{V})$  and kinetic energy  $\mathbf{E}_k(\mathbf{V})$  on speed  $\mathbf{V}$ :

$$M(V) = \frac{M_0}{\sqrt{1 - \frac{V^2}{V_{\text{кр}}^2}}} \quad (146)$$

$$P(V) = \frac{M_0 \cdot V}{\sqrt{1 - \frac{V^2}{V_{\text{кр}}^2}}} \quad (147)$$

$$E_k(V) = M_0 \cdot V_{\text{кр}}^2 \cdot \left( \frac{1}{\sqrt{1 - \frac{V^2}{V_{\text{кр}}^2}}} - 1 \right) \quad (148)$$

#### III.1.4. The formula of the relationship of the mass of the moving body with speed when the transition coefficient is $\beta > 1$

In the event that the value  $\beta$  of the transition coefficient  $\beta$  is within the range  $\beta > 1$ , based on the formulas (145)-(148) taking into account equation (55) according to the function  $\mathbf{f}(\mathbf{V})_>$ , the mass  $\mathbf{M}(\mathbf{V})_>$ , the momentum  $\mathbf{P}(\mathbf{V})_>$ , the kinetic energy  $\mathbf{E}_k(\mathbf{V})_>$  of the body moving with speed  $\mathbf{V}$  can be written:

$$f(V)_> = \frac{1}{\sqrt{1 - \frac{V^2}{V_{\text{кр}1}^2}}} \quad (149)$$

$$M(V)_> = \frac{M_0}{\sqrt{1 - \frac{V^2}{V_{\text{кр}1}^2}}} \quad (150)$$

$$P(V)_> = \frac{M_0 \cdot V}{\sqrt{1 - \frac{V^2}{V_{\text{кр}1}^2}}} \quad (151)$$

$$E_{\kappa}(V)_{>} = M_0 \cdot v_{\text{хкр}1}^2 \cdot \left( \frac{1}{\sqrt{1 - \frac{V^2}{v_{\text{хкр}1}^2}}} - 1 \right) \quad (152)$$

**III.1.4.1. The verification of the correct choice of formula (145),  
when  $\beta > 1$  (for examples № 1 and № 2)**

First, rewrite the formulas (128)-(136), taking into account formula (55) and (149)-(152):

$$\frac{M_{01} \cdot v_{21\text{хн}}}{\sqrt{1 - \frac{v_{21\text{хн}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02} \cdot v_{22\text{хн}}}{\sqrt{1 - \frac{v_{22\text{хн}}^2}{v_{\text{хкр}1}^2}}} = \frac{M_{01} \cdot v_{21\text{хк}}}{\sqrt{1 - \frac{v_{21\text{хк}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02} \cdot v_{22\text{хк}}}{\sqrt{1 - \frac{v_{22\text{хк}}^2}{v_{\text{хкр}1}^2}}} \quad (153)$$

$$\frac{M_{01}}{\sqrt{1 - \frac{v_{21\text{хн}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02}}{\sqrt{1 - \frac{v_{22\text{хн}}^2}{v_{\text{хкр}1}^2}}} = \frac{M_{01}}{\sqrt{1 - \frac{v_{21\text{хк}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02}}{\sqrt{1 - \frac{v_{22\text{хк}}^2}{v_{\text{хкр}1}^2}}} \quad (154)$$

$$\frac{M_{01} \cdot v_{11\text{хн}}}{\sqrt{1 - \frac{v_{11\text{хн}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02} \cdot v_{12\text{хн}}}{\sqrt{1 - \frac{v_{12\text{хн}}^2}{v_{\text{хкр}1}^2}}} = \frac{M_{01} \cdot v_{11\text{хк}}}{\sqrt{1 - \frac{v_{11\text{хк}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02} \cdot v_{12\text{хк}}}{\sqrt{1 - \frac{v_{12\text{хк}}^2}{v_{\text{хкр}1}^2}}} \quad (155)$$

$$\frac{M_{01}}{\sqrt{1 - \frac{v_{11\text{хн}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02}}{\sqrt{1 - \frac{v_{12\text{хн}}^2}{v_{\text{хкр}1}^2}}} = \frac{M_{01}}{\sqrt{1 - \frac{v_{11\text{хк}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02}}{\sqrt{1 - \frac{v_{12\text{хк}}^2}{v_{\text{хкр}1}^2}}} \quad (156)$$

$$\frac{M_{01} \cdot v_{21\text{ун}}}{\sqrt{1 - \frac{v_{21\text{ун}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02} \cdot v_{22\text{ун}}}{\sqrt{1 - \frac{v_{22\text{ун}}^2}{v_{\text{хкр}1}^2}}} = \frac{M_{01} \cdot v_{21\text{ук}}}{\sqrt{1 - \frac{v_{21\text{ук}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02} \cdot v_{22\text{ук}}}{\sqrt{1 - \frac{v_{22\text{ук}}^2}{v_{\text{хкр}1}^2}}} \quad (157)$$

$$\frac{M_{01}}{\sqrt{1 - \frac{v_{21\text{ун}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02}}{\sqrt{1 - \frac{v_{22\text{ун}}^2}{v_{\text{хкр}1}^2}}} = \frac{M_{01}}{\sqrt{1 - \frac{v_{21\text{ук}}^2}{v_{\text{хкр}1}^2}}} + \frac{M_{02}}{\sqrt{1 - \frac{v_{22\text{ук}}^2}{v_{\text{хкр}1}^2}}} \quad (158)$$

$$\begin{aligned}
& \frac{M_{01} \cdot V}{\sqrt{1 - \frac{v_{11yH}^2 + V^2}{v_{xkp1}^2}}} + \frac{M_{02} \cdot V}{\sqrt{1 - \frac{v_{12yH}^2 + V^2}{v_{xkp1}^2}}} = \\
& = \frac{M_{01} \cdot V}{\sqrt{1 - \frac{v_{11yK}^2 + V^2}{v_{xkp1}^2}}} + \frac{M_{02} \cdot V}{\sqrt{1 - \frac{v_{12yK}^2 + V^2}{v_{xkp1}^2}}} \quad (159)
\end{aligned}$$

$$\begin{aligned}
& \frac{M_{01} \cdot v_{11yH}}{\sqrt{1 - \frac{v_{11yH}^2 + V^2}{v_{xkp1}^2}}} + \frac{M_{02} \cdot v_{12yH}}{\sqrt{1 - \frac{v_{12yH}^2 + V^2}{v_{xkp1}^2}}} = \\
& = \frac{M_{01} \cdot v_{11yK}}{\sqrt{1 - \frac{v_{11yK}^2 + V^2}{v_{xkp1}^2}}} + \frac{M_{02} \cdot v_{12yK}}{\sqrt{1 - \frac{v_{12yK}^2 + V^2}{v_{xkp1}^2}}} \quad (160)
\end{aligned}$$

$$\begin{aligned}
& \frac{M_{01}}{\sqrt{1 - \frac{v_{11yH}^2 + V^2}{v_{xkp1}^2}}} + \frac{M_{02}}{\sqrt{1 - \frac{v_{12yH}^2 + V^2}{v_{xkp1}^2}}} = \\
& = \frac{M_{01}}{\sqrt{1 - \frac{v_{11yK}^2 + V^2}{v_{xkp1}^2}}} + \frac{M_{02}}{\sqrt{1 - \frac{v_{12yK}^2 + V^2}{v_{xkp1}^2}}} \quad (161)
\end{aligned}$$

Where, on the basis of the formulas (55) and (137)-(144):

$$v_{11xH} = \frac{v_{21xH} + V}{1 + \frac{V \cdot v_{21xH}}{v_{xkp1}^2}} \quad (162)$$

$$v_{12xH} = \frac{v_{22xH} + V}{1 + \frac{V \cdot v_{22xH}}{v_{xkp1}^2}} \quad (163)$$

$$v_{11xK} = \frac{v_{21xK} + V}{1 + \frac{V \cdot v_{21xK}}{v_{xkp1}^2}} \quad (164)$$

$$v_{12xK} = \frac{v_{22xK} + V}{1 + \frac{V \cdot v_{22xK}}{v_{xkp1}^2}} \quad (165)$$

$$v_{11y_H} = v_{21y_H} \cdot \sqrt{1 - \frac{V^2}{v_{xkp1}^2}} \quad (166)$$

$$v_{11y_K} = v_{21y_K} \cdot \sqrt{1 - \frac{V^2}{v_{xkp1}^2}} \quad (167)$$

$$v_{12y_H} = v_{22y_H} \cdot \sqrt{1 - \frac{V^2}{v_{xkp1}^2}} \quad (168)$$

$$v_{12y_K} = v_{22y_K} \cdot \sqrt{1 - \frac{V^2}{v_{xkp1}^2}} \quad (169)$$

Suppose that  $M_{o1} = 1$  ,  $M_{o2} = 0,5$  ,  $V / v_{xkp1} = 0,5$  ,  $v_{21x_H} / v_{xkp1} =$   
 $= v_{21y_H} / v_{xkp1} = 0,9$  ,  $v_{22x_H} / v_{xkp1} = v_{22y_H} / v_{xkp1} = 0,6$  .

Then numerical calculations gave the following results for example № 1:

I. In the moving reference system  $O_2x_2y_2z_2$ :

1) body 1 was:

Object	Period of time	Name value	Value
Body 1	Before the collision	speed $v_{21x_H} / v_{xkp1}$	0,9
		mass $M_{21H}$	2,294157338706
		momentum $P_{21H} / v_{xkp1}$	2,064741604835
		kinetic energy $E_{K21H} / v_{xkp1}^2$	1,294157338706
	After the collision	speed $v_{21x_K} / v_{xkp1}$	0,7360143377
		mass $M_{21K}$	1,477179174242
		momentum $P_{21K} / v_{xkp1}$	1,087225051595
		kinetic energy $E_{K21K} / v_{xkp1}^2$	0,477179174242

2) body 2 was:

Object	Period of time	Name value	Value
Body 2	Before the collision	speed $v_{22xH} / v_{xkp1}$	0,6
		mass $M_{22H}$	0,625
		momentum $P_{22H} / v_{xkp1}$	0,375
		kinetic energy $E_{K22H} / v_{xkp1}^2$	0,125
	After the collision	speed $v_{22xK} / v_{xkp1}$	0,937959108239
		mass $M_{22K}$	1,441978164463
		momentum $P_{22K} / v_{xkp1}$	1,35251655324
		kinetic energy $E_{K22K} / v_{xkp1}^2$	0,941978164463

3) the system of bodies 1 and 2 was:

Object	Period of time	Name value	Value
System of bodies 1 and 2	Before the collision	mass ( $M_{21H} + M_{22H}$ )	2,919157338706
		momentum $(P_{21H} + P_{22H}) / v_{xkp1}$	2,439741604835
		kinetic energy $E_{K22H} / v_{xkp1}^2$	1,419157338706
	After the collision	mass ( $M_{21K} + M_{22K}$ )	2,919157338706
		momentum $(P_{21K} + P_{22K}) / v_{xkp1}$	2,439741604835
		kinetic energy $(E_{K21K} + E_{K22K}) / v_{xkp1}^2$	1,419157338706

II. In the fixed reference system  $O_1x_1y_1z_1$ :

1) body 1 was:

Object	Period of time	Name value	Value
Body 1	Before the collision	speed $v_{11xH} / v_{xkp1}$	0,965517241379
		mass $M_{11H}$	3,841143835489
		momentum $P_{11H} / v_{xkp1}$	3,708690599782
		kinetic energy $E_{K11H} / v_{xkp1}^2$	2,841143835489
	After the collision	speed $v_{11xK} / v_{xkp1}$	0,903514517939
		mass $M_{11K}$	2,333409263988
		momentum $P_{11K} / v_{xkp1}$	2,108269146306
		kinetic energy $E_{K11K} / v_{xkp1}^2$	1,333409263988

2) body 2 was:

Object	Period of time	Name value	Value
Body 2	Before the collision	speed $v_{12xH} / v_{xkp1}$	0,846153846154
		mass $M_{12H}$	0,938194187433
		momentum $P_{12H} / v_{xkp1}$	0,793856620136
		kinetic energy $E_{K12H} / v_{xkp1}^2$	0,438194187433
	After the collision	speed $v_{12xK} / v_{xkp1}$	0,978882996844
		mass $M_{12K}$	2,445928758933
		momentum $P_{12K} / v_{xkp1}$	2,394278073612
		kinetic energy $E_{K12K} / v_{xkp1}^2$	1,945928758933

3) the system of the bodies 1 and 2 was:

Object	Period of time	Name value	Value
System of bodies 1 and 2	Before the collision	mass ( $M_{11H} + M_{12H}$ )	4,779338022922
		momentum $(P_{11H} + P_{12H}) / v_{xkp1}$	4,502547219918
		kinetic energy $(E_{K11H} + E_{K12H}) / v_{xkp1}^2$	3,279338022922
	After the collision	mass ( $M_{11K} + M_{12K}$ )	4,779338022922
		momentum $(P_{11K} + P_{12K}) / v_{xkp1}$	4,502547219918
		kinetic energy $(E_{K11K} + E_{K12K}) / v_{xkp1}^2$	3,279338022922

For example № 2, the numerical calculations give the following results:

I. In the moving reference system  $O_2x_2y_2z_2$ :

1) body 1 was:

Object	Period of time	Name value	Value
Body 1	Before the collision	speed $v_{21yH} / v_{xkp1}$	0,9
		mass $M_{21H}$	2,294157338706
		momentum $P_{21H} / v_{xkp1}$	2,064741604835
		kinetic energy $E_{K21H} / v_{xkp1}^2$	1,294157338706
	After the collision	speed $v_{21yK} / v_{xkp1}$	0,7360143377
		mass $M_{21K}$	1,477179174242
		momentum $P_{21K} / v_{xkp1}$	1,087225051595
		kinetic energy $E_{K21K} / v_{xkp1}^2$	0,477179174242

2) body 2 was:

Object	Period of time	Name value	Value
Body 2	Before the collision	speed $v_{22vH} / v_{xkp1}$	0,6
		mass $M_{22H}$	0,625
		momentum $P_{22H} / v_{xkp1}$	0,375
		kinetic energy $E_{K22H} / v_{xkp1}^2$	0,125
	After the collision	mass $v_{22vK} / v_{xkp1}$	0,937959108239
		macca $M_{22K}$	1,441978164463
		momentum $P_{22K} / v_{xkp1}$	1,35251655324
		kinetic energy $E_{K22K} / v_{xkp1}^2$	0,941978164463

3) the system of bodies 1 and 2 was:

Object	Period of time	Name value	Value
System of bodies 1 and 2	Before the collision	mass ( $M_{21H} + M_{22H}$ )	2,919157338706
		momentum $(P_{21H} + P_{22H}) / v_{xkp1}$	2,439741604835
		kinetic energy $E_{K22H} / v_{xkp1}^2$	1,419157338706
	After the collision	mass ( $M_{21K} + M_{22K}$ )	2,919157338706
		momentum $(P_{21K} + P_{22K}) / v_{xkp1}$	2,439741604835
		kinetic energy $(E_{K21K} + E_{K22K}) / v_{xkp1}^2$	1,419157338706

II. In the fixed reference system  $O_1x_1y_1z_1$ :

1) body 1 was:

Object	Period of time	Name value	Value
Body 1	Before the collision	projections of the speed $v_{11xH} / v_{xkp1}$	0,5
		projections of the speed $v_{11yH} / v_{xkp1}$	0,779422863406
		mass $M_{11H}$	2,64906471413
		projection of the momentum $P_{11xH} / v_{xkp1}$	1,324532357065
		projection of the momentum $P_{11yH} / v_{xkp1}$	2,064741604835
		kinetic energy $E_{K11H} / v_{xkp1}^2$	1,64906471413
	After the collision	projections of the speed $v_{11xK} / v_{xkp1}$	0,5
		projections of the speed $v_{11yK} / v_{xkp1}$	0,637407113998
		mass $M_{11K}$	1,70569958778
		projection of the momentum $P_{11xK} / v_{xkp1}$	0,85284979389
		projection of the momentum $P_{11yK} / v_{xkp1}$	1,087225051595
		kinetic energy $E_{K11K} / v_{xkp1}^2$	0,70569958778



2) body 2 was:

Object	Period of time	Name value	Value
Body 2	Before the collision	projections of the speed $v_{12xH} / v_{xkp1}$	0,5
		projections of the speed $v_{12yH} / v_{xkp1}$	0,519615242271
		mass $M_{12H}$	0,721687836487
		projection of the momentum $P_{12xH} / v_{xkp1}$	0,360843918244
		projection of the momentum $P_{12yH} / v_{xkp1}$	0,375
		kinetic energy $E_{K12H} / v_{xkp1}^2$	0,221687836487
	After the collision	projections of the speed $v_{12xK} / v_{xkp1}$	0,5
		projections of the speed $v_{12yK} / v_{xkp1}$	0,812296415446
		mass $M_{12K}$	1,665052962837
		projection of the momentum $P_{12xK} / v_{xkp1}$	0,832526481418
		projection of the momentum $P_{12yK} / v_{xkp1}$	1,35251655324
		kinetic energy $E_{K12K} / v_{xkp1}^2$	1,165052962837

3) the system of bodies 1 and 2 was:

Object	Period of time	Name value	Value
System of bodies 1 and 2	Before the collision	mass ( $M_{11H} + M_{12H}$ )	3,370752550617
		projection of the momentum ( $P_{11xH} + P_{12xH}$ ) / $v_{xkp1}$	1,685376275309
		projection of the momentum ( $P_{11yH} + P_{12yH}$ ) / $v_{xkp1}$	2,439741604835
		kinetic energy ( $E_{k11H} + E_{k12H}$ ) / $v_{xkp1}^2$	1,870752550617
	After the collision	mass ( $M_{11K} + M_{12K}$ )	3,370752550617
		projection of the momentum ( $P_{11xK} + P_{12xK}$ ) / $v_{xkp1}$	1,685376275309
		projection of the momentum ( $P_{11yK} + P_{12yK}$ ) / $v_{xkp1}$	2,439741604835
		kinetic energy ( $E_{k11K} + E_{k12K}$ ) / $v_{xkp1}^2$	1,870752550617

According to the results of the calculation, we come to the following conclusion: in examples № 1 and № 2 in the moving  $O_2x_2y_2z_2$  and fixed  $O_1x_1y_1z_1$  reference systems before and after the collision, the mass, the momentum and the kinetic energy of the mechanical system of bodies 1 and 2 remain unchanged.

Consequently, the formulas (145)-(148), when the transition coefficient is  $\beta > 1$ , meet the requirements of the system of equations (128)-(136).

### 3.1.5. The formula of the relationship of the mass of the moving body and speed, when the transition coefficient is $0 < \beta < 1$

Where the value of the transition coefficient  $\beta$  is in the range  $0 < \beta < 1$ , based on formulas (145)-(148) taking into account equation (56) can be written according to function  $f(\mathbf{V})_<$ , mass  $M(\mathbf{V})_<$ , momentum  $\mathbf{P}(\mathbf{V})_<$ , kinetic energy  $E_k(\mathbf{V})_<$  of the body moving with speed  $\mathbf{V}$ , you can write:

$$f(V)_{<} = \frac{1}{\sqrt{1 + \frac{V^2}{v_{\text{кр}2}^2}}} \quad (170)$$

$$M(V)_{<} = \frac{M_0}{\sqrt{1 + \frac{V^2}{v_{\text{кр}2}^2}}} \quad (171)$$

$$P(V)_{<} = \frac{M_0 \cdot V}{\sqrt{1 + \frac{V^2}{v_{\text{кр}2}^2}}} \quad (172)$$

$$E_{\kappa}(V)_{<} = M_0 \cdot v_{\text{кр}2}^2 \cdot \left( 1 - \frac{1}{\sqrt{1 + \frac{V^2}{v_{\text{кр}2}^2}}} \right) \quad (173)$$

**3.1.5.1. The verification of the correct choice of formula (145),  
when  $0 < \beta < 1$  (for examples № 1 and № 2)**

First rewrite formulas (128)-(136), taking into the account formulas (56) and (170)-(173):

$$\frac{M_{01} \cdot v_{21\text{XH}}}{\sqrt{1 + \frac{v_{21\text{XH}}^2}{v_{\text{кр}2}^2}}} + \frac{M_{02} \cdot v_{22\text{XH}}}{\sqrt{1 + \frac{v_{22\text{XH}}^2}{v_{\text{кр}2}^2}}} = \frac{M_{01} \cdot v_{21\text{XK}}}{\sqrt{1 + \frac{v_{21\text{XK}}^2}{v_{\text{кр}2}^2}}} + \frac{M_{02} \cdot v_{22\text{XK}}}{\sqrt{1 + \frac{v_{22\text{XK}}^2}{v_{\text{кр}2}^2}}} \quad (174)$$

$$\frac{M_{01}}{\sqrt{1 + \frac{v_{21\text{XH}}^2}{v_{\text{кр}2}^2}}} + \frac{M_{02}}{\sqrt{1 + \frac{v_{22\text{XH}}^2}{v_{\text{кр}2}^2}}} = \frac{M_{01}}{\sqrt{1 + \frac{v_{21\text{XK}}^2}{v_{\text{кр}2}^2}}} + \frac{M_{02}}{\sqrt{1 + \frac{v_{22\text{XK}}^2}{v_{\text{кр}2}^2}}} \quad (175)$$

$$\frac{M_{01} \cdot v_{11\text{XH}}}{\sqrt{1 + \frac{v_{11\text{XH}}^2}{v_{\text{кр}2}^2}}} + \frac{M_{02} \cdot v_{12\text{XH}}}{\sqrt{1 + \frac{v_{12\text{XH}}^2}{v_{\text{кр}2}^2}}} = \frac{M_{01} \cdot v_{11\text{XK}}}{\sqrt{1 + \frac{v_{11\text{XK}}^2}{v_{\text{кр}2}^2}}} + \frac{M_{02} \cdot v_{12\text{XK}}}{\sqrt{1 + \frac{v_{12\text{XK}}^2}{v_{\text{кр}2}^2}}} \quad (176)$$

$$\frac{M_{01}}{\sqrt{1 + \frac{v_{11xH}^2}{v_{xkp2}^2}}} + \frac{M_{02}}{\sqrt{1 + \frac{v_{12xH}^2}{v_{xkp2}^2}}} = \frac{M_{01}}{\sqrt{1 + \frac{v_{11xK}^2}{v_{xkp2}^2}}} + \frac{M_{02}}{\sqrt{1 + \frac{v_{12xK}^2}{v_{xkp2}^2}}} \quad (177)$$

$$\frac{M_{01} \cdot v_{21yH}}{\sqrt{1 + \frac{v_{21yH}^2}{v_{xkp2}^2}}} + \frac{M_{02} \cdot v_{22yH}}{\sqrt{1 + \frac{v_{22yH}^2}{v_{xkp2}^2}}} = \frac{M_{01} \cdot v_{21yK}}{\sqrt{1 + \frac{v_{21yK}^2}{v_{xkp2}^2}}} + \frac{M_{02} \cdot v_{22yK}}{\sqrt{1 + \frac{v_{22yK}^2}{v_{xkp2}^2}}} \quad (178)$$

$$\frac{M_{01}}{\sqrt{1 + \frac{v_{21yH}^2}{v_{xkp2}^2}}} + \frac{M_{02}}{\sqrt{1 + \frac{v_{22yH}^2}{v_{xkp2}^2}}} = \frac{M_{01}}{\sqrt{1 + \frac{v_{21yK}^2}{v_{xkp2}^2}}} + \frac{M_{02}}{\sqrt{1 + \frac{v_{22yK}^2}{v_{xkp2}^2}}} \quad (179)$$

$$\begin{aligned} & \frac{M_{01} \cdot V}{\sqrt{1 + \frac{v_{11yH}^2 + V^2}{v_{xkp2}^2}}} + \frac{M_{02} \cdot V}{\sqrt{1 + \frac{v_{12yH}^2 + V^2}{v_{xkp2}^2}}} = \\ & = \frac{M_{01} \cdot V}{\sqrt{1 + \frac{v_{11yK}^2 + V^2}{v_{xkp2}^2}}} + \frac{M_{02} \cdot V}{\sqrt{1 + \frac{v_{12yK}^2 + V^2}{v_{xkp2}^2}}} \end{aligned} \quad (180)$$

$$\begin{aligned} & \frac{M_{01} \cdot v_{11yH}}{\sqrt{1 + \frac{v_{11yH}^2 + V^2}{v_{xkp2}^2}}} + \frac{M_{02} \cdot v_{12yH}}{\sqrt{1 + \frac{v_{12yH}^2 + V^2}{v_{xkp2}^2}}} = \\ & = \frac{M_{01} \cdot v_{11yK}}{\sqrt{1 + \frac{v_{11yK}^2 + V^2}{v_{xkp2}^2}}} + \frac{M_{02} \cdot v_{12yK}}{\sqrt{1 + \frac{v_{12yK}^2 + V^2}{v_{xkp2}^2}}} \end{aligned} \quad (181)$$

$$\begin{aligned} & \frac{M_{01}}{\sqrt{1 + \frac{v_{11yH}^2 + V^2}{v_{xkp1}^2}}} + \frac{M_{02}}{\sqrt{1 + \frac{v_{12yH}^2 + V^2}{v_{xkp1}^2}}} = \\ & = \frac{M_{01}}{\sqrt{1 + \frac{v_{11yK}^2 + V^2}{v_{xkp1}^2}}} + \frac{M_{02}}{\sqrt{1 + \frac{v_{12yK}^2 + V^2}{v_{xkp1}^2}}} \end{aligned} \quad (182)$$

Where, on the basis of the formulas (56) and (137)-(144):

$$V_{11xH} = \frac{V_{21xH} + V}{1 - \frac{V \cdot V_{21xH}}{V_{xkp2}^2}} \quad (183)$$

$$V_{12xH} = \frac{V_{22xH} + V}{1 - \frac{V \cdot V_{22xH}}{V_{xkp2}^2}} \quad (184)$$

$$V_{11xK} = \frac{V_{21xK} + V}{1 - \frac{V \cdot V_{21xK}}{V_{xkp2}^2}} \quad (185)$$

$$V_{12xK} = \frac{V_{22xK} + V}{1 - \frac{V \cdot V_{22xK}}{V_{xkp2}^2}} \quad (186)$$

$$V_{11yH} = V_{21yH} \cdot \sqrt{1 + \frac{V^2}{V_{xkp1}^2}} \quad (187)$$

$$V_{11yK} = V_{21yK} \cdot \sqrt{1 + \frac{V^2}{V_{xkp2}^2}} \quad (188)$$

$$V_{12yH} = V_{22yH} \cdot \sqrt{1 + \frac{V^2}{V_{xkp2}^2}} \quad (189)$$

$$V_{12yK} = V_{22yK} \cdot \sqrt{1 + \frac{V^2}{V_{xkp2}^2}} \quad (190)$$

Suppose that  $M_{01} = 1$ ,  $M_{02} = 0,5$ ,  $V / v_{xkp2} = 0,5$ ,  $v_{21xH} / v_{xkp2} = v_{21yH} / v_{xkp2} = 0,9$ ,  $v_{22xH} / v_{xkp2} = v_{22yH} / v_{xkp2} = 0,6$ .

Then the numerical calculations give the following results for example № 1:

I. In the moving reference system  $O_2x_2y_2z_2$ :

1) body 1 was:

Object	Period of time	Name value	Value
Body 1	Before the collision	speed $v_{21xH} / v_{xkp2}$	0,9
		mass $M_{21H}$	0,743294146247
		momentum $P_{21H} / v_{xkp2}$	0,668964731622
		kinetic energy $E_{K21H} / v_{xkp2}^2$	0,256705853753
	After the collision	speed $v_{21xK} / v_{xkp2}$	0,691099932748
		mass $M_{21K}$	0,822656908881
		momentum $P_{21K} / v_{xkp2}$	0,568538134403
		kinetic energy $E_{K21K} / v_{xkp2}^2$	0,177343091119

2) body 2 was:

Object	Period of time	Name value	Value
Body 2	Before the collision	speed $v_{22xH} / v_{xkp2}$	0,6
		mass $M_{22H}$	0,428746462856
		momentum $P_{22H} / v_{xkp2}$	0,257247877714
		kinetic energy $E_{K22H} / v_{xkp2}^2$	0,071253537144
	After the collision	speed $v_{22xK} / v_{xkp2}$	1,023729712365
		mass $M_{22K}$	0,349383700222
		momentum $P_{22K} / v_{xkp2}$	0,357674474934
		kinetic energy $E_{K22K} / v_{xkp2}^2$	0,150616299778

3) the system of bodies 1 and 2 was:

Object	Period of time	Name value	Value
System of bodies 1 and 2	Before the collision	mass ( $M_{21H} + M_{22H}$ )	1,172040609103
		momentum $(P_{21H} + P_{22H}) / v_{xkp2}$	0,926212609336
		kinetic energy $E_{K22H} / v_{xkp2}^2$	0,327959390897
	After the collision	mass ( $M_{21K} + M_{22K}$ )	1,172040609103
		momentum $(P_{21K} + P_{22K}) / v_{xkp2}$	0,926212609336
		kinetic energy $(E_{K21K} + E_{K22K}) / v_{xkp2}^2$	0,327959390897

II. In the fixed reference system  $O_1x_1y_1z_1$ :

1) body 1 was:

Object	Period of time	Name value	Value
Body 1	Before the collision	speed $v_{11xH} / v_{xkp2}$	2,545454545455
		mass $M_{11H}$	0,365652372423
		momentum $P_{11H} / v_{xkp2}$	0,93075149344
		kinetic energy $E_{K11H} / v_{xkp2}^2$	0,634347627577
	After the collision	speed $v_{11xK} / v_{xkp2}$	1,820001331727
		mass $M_{11K}$	0,481548724902
		momentum $P_{11K} / v_{xkp2}$	0,876419320614
		kinetic energy $E_{K11K} / v_{xkp2}^2$	0,518451275098

2) body 2 was:

Object	Period of time	Name value	Value
Body 2	Before the collision	speed $v_{12xH} / v_{xkp2}$	1,571428571429
		mass $M_{12H}$	0,268437746097
		momentum $P_{12H} / v_{xkp2}$	0,421830743866
		kinetic energy $E_{K12H} / v_{xkp2}^2$	0,231562253903
	After the collision	speed $v_{12xK} / v_{xkp2}$	3,121532492927
		mass $M_{12K}$	0,152541393617
		momentum $P_{12K} / v_{xkp2}$	0,476162916693
		kinetic energy $E_{K12K} / v_{xkp2}^2$	0,347458606383

3) the system of bodies 1 and 2 was:

Object	Period of time	Name value	Value
System of bodies 1 and 2	Before the collision	mass $(M_{11H} + M_{12H})$	0,63409011852
		momentum $(P_{11H} + P_{12H}) / v_{xkp2}$	1,352582237306
		kinetic energy $(E_{K11H} + E_{K12H}) / v_{xkp2}^2$	0,86590988148
	After the collision	mass $(M_{11K} + M_{12K})$	0,63409011852
		momentum $(P_{11K} + P_{12K}) / v_{xkp2}$	1,352582237306
		kinetic energy $(E_{K11K} + E_{K12K}) / v_{xkp2}^2$	0,86590988148

For example № 2 the numerical calculations give the following results:

I. In the moving reference system  $O_2x_2y_2z_2$ :

1) body 1 was:

Object	Period of time	Name value	Value
Body 1	Before the collision	speed $v_{21vH} / v_{xkp2} = 0,9$	0,9
		mass $M_{21H}$	0,743294146247
		momentum $P_{21H} / v_{xkp2}$	0,668964731622
		kinetic energy $E_{K21H} / v_{xkp2}^2$	0,256705853753
	After the collision	speed $v_{21vK} / v_{xkp2}$	0,691099932748
		mass $M_{21K}$	0,822656908881
		momentum $P_{21K} / v_{xkp2}$	0,568538134403
		kinetic energy $E_{K21K} / v_{xkp2}^2$	0,177343091119

2) body 2 was:

Object	Period of time	Name value	Value
Body 2	Before the collision	speed $v_{22vH} / v_{xkp2}$	0,6
		mass $M_{22H}$	0,428746462856
		momentum $P_{22H} / v_{xkp2}$	0,257247877714
		kinetic energy $E_{K22H} / v_{xkp2}^2$	0,071253537144
	After the collision	speed $v_{22vK} / v_{xkp2}$	1,023729712365
		mass $M_{22K}$	0,349383700222
		momentum $P_{22K} / v_{xkp2}$	0,357674474934
		kinetic energy $E_{K22K} / v_{xkp2}^2$	0,150616299778

3) the system of bodies 1 and 2 was:

Object	Period of time	Name value	Value
System of bodies 1 and 2	Before the collision	mass ( $M_{21H} + M_{22H}$ )	1,172040609103
		momentum $(P_{21H} + P_{22H}) / v_{xkp2}$	0,926212609336
		kinetic energy $E_{K22H} / v_{xkp2}^2$	0,327959390897
	After the collision	mass ( $M_{21K} + M_{22K}$ )	1,172040609103
		momentum $(P_{21K} + P_{22K}) / v_{xkp2}$	0,926212609336
		kinetic energy $(E_{K21K} + E_{K22K}) / v_{xkp2}^2$	0,327959390897



II. In the fixed reference system  $O_1x_1y_1z_1$ :

1) body 1 was:

Object	Period of time	Name value	Value
Body 1	Before the collision	projections of the speed $v_{11xH} / v_{xkp2}$	0,5
		projections of the speed $v_{11yH} / v_{xkp2}$	1,006230589875
		mass $M_{11H}$	0,664822495315
		projection of the momentum $P_{11xH} / v_{xkp2}$	0,332411247657
		projection of the momentum $P_{11yH} / v_{xkp2}$	0,668964731622
		kinetic energy $E_{K11H} / v_{xkp2}^2$	0,335177504685
	After the collision	projections of the speed $v_{11xK} / v_{xkp2}$	0,5
		projections of the speed $v_{11yK} / v_{xkp2}$	0,772673214435
		mass $M_{11K}$	0,735806708167
		projection of the momentum $P_{11xK} / v_{xkp2}$	0,367903354084
		projection of the momentum $P_{11yK} / v_{xkp2}$	0,568538134403
		kinetic energy $E_{K11K} / v_{xkp2}^2$	0,264193291833

2) body 2 was:

Object	Period of time	Name value	Value
Body 2	Before the collision	projections of the speed $v_{12xH} / v_{xkp2}$	0,5
		projections of the speed $v_{12yH} / v_{xkp2}$	0,67082039325
		mass $M_{12H}$	0,383482494424
		projection of the momentum $P_{12xH} / v_{xkp2}$	0,191741247212
		projection of the momentum $P_{12yH} / v_{xkp2}$	0,257247877714
		kinetic energy $E_{K12H} / v_{xkp2}^2$	0,116517505576
	After the collision	projections of the speed $v_{12xK} / v_{xkp2}$	0,5
		projections of the speed $v_{12yK} / v_{xkp2}$	1,144564613718
		mass $M_{12K}$	0,312498281571
		projection of the momentum $P_{12xK} / v_{xkp2}$	0,156249140785
		projection of the momentum $P_{12yK} / v_{xkp2}$	0,357674474934
		kinetic energy $E_{K12K} / v_{xkp2}^2$	0,187501718429

3) the system of bodies 1 and 2 was:

Object	Period of time	Name value	Value
System of bodies 1 and 2	Before the collision	mass ( $M_{11H} + M_{12H}$ )	1,048304989738
		projection of the momentum ( $P_{11xH} + P_{12xH}$ ) / $v_{xkp2}$	0,524152494869
		projection of the momentum ( $P_{11yH} + P_{12yH}$ ) / $v_{xkp2}$	0,926212609336
		kinetic energy ( $E_{k11H} + E_{k12H}$ ) / $v_{xkp2}^2$	0,451695010262
	After the collision	mass ( $M_{11K} + M_{12K}$ )	1,048304989738
		projection of the momentum ( $P_{11xK} + P_{12xK}$ ) / $v_{xkp2}$	0,524152494869
		projection of the momentum ( $P_{11yK} + P_{12yK}$ ) / $v_{xkp2}$	0,926212609336
		kinetic energy ( $E_{k11K} + E_{k12K}$ ) / $v_{xkp2}^2$	0,451695010262

According to the results of the calculation we reach the following conclusion: in examples № 1 and № 2 in the moving  $O_2x_2y_2z_2$  and fixed  $O_1x_1y_1z_1$  reference systems before and after the collision the mass, the momentum and the kinetic energy of the mechanical system of bodies 1 and 2 remain unchanged.

Consequently, formulas (170)-(173), when the transition coefficient is  $0 < \beta < 1$ , meet the requirements of the system of equations (128)-(136).

### 3.1.6. Comparing formulas (150)-(152) with formulas (171)-(173)

About dependences (150)-(152):

$$M(V)_> = \frac{M_0}{\sqrt{1 - \frac{V^2}{v_{xkp1}^2}}} \quad (150)$$

$$P(V)_> = \frac{M_0 \cdot V}{\sqrt{1 - \frac{V^2}{v_{xkp1}^2}}} \quad (151)$$

$$E_k(V)_> = M_0 \cdot v_{xkp1}^2 \cdot \left( \frac{1}{\sqrt{1 - \frac{V^2}{v_{xkp1}^2}}} - 1 \right) \quad (152)$$

of the moving body, mass  $\mathbf{M(V)}_>$ , momentum  $\mathbf{P(V)}_>$  and kinetic energy  $\mathbf{E}_k(\mathbf{V})_>$  of the speed  $\mathbf{V}$ , when the transition coefficient  $\beta > 1$ , you can say the following:

Speed $\mathbf{V}$	Mass $\mathbf{M(V)}_>$	Momentum $\mathbf{P(V)}_>$	Kinetic energy $\mathbf{E}_k(\mathbf{V})_>$
$V \ll v_{xkp1}$	$M_0$	$M_0 \cdot V$	$\frac{M_0 \cdot V^2}{2}$
$V < v_{xkp1}$	have valid values	have valid values	have valid values
$V = v_{xkp1}$	$\infty$	$\infty$	$\infty$
$V > v_{xkp1}$	do not have valid values	do not have valid values	do not have valid values

Similarly, on the dependence (171)-(173):

$$M(V)_< = \frac{M_0}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \quad (171)$$

$$P(V)_< = \frac{M_0 \cdot V}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \quad (172)$$

$$E_k(V)_< = M_0 \cdot v_{xkp2}^2 \cdot \left( 1 - \frac{1}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \right) \quad (173)$$

of the moving body with mass  $\mathbf{M(V)}_<$ , momentum  $\mathbf{P(V)}_<$ , and kinetic energy  $\mathbf{E}_k(\mathbf{V})_<$  of speed  $\mathbf{V}$ , when the transition coefficient  $0 < \beta < 1$ , you can say the following:

Speed $V$	Mass $M(V)_{<}$	Momentum $P(V)_{<}$	Kinetic energy $E_k(V)_{<}$
$V \ll v_{xkp2}$	$M_0$	$M_0 \cdot V$	$\frac{M_0 \cdot V^2}{2}$
$V < v_{xkp2}$	have valid values	have valid values	have valid values
$V = v_{xkp2}$	$\frac{M_0}{\sqrt{2}}$	$\frac{M_0 \cdot v_{xkp2}}{\sqrt{2}}$	$M_0 \cdot v_{xkp2}^2 \cdot \left(1 - \frac{1}{\sqrt{2}}\right)$
$V > v_{xkp2}$	have valid values	have valid values	have valid values
$V = \infty$	seeks to zero	$M_0 \cdot v_{xkp2}$	$M_0 \cdot v_{xkp2}^2$

As can be seen from the comparison, two possible values of the range of the transition coefficient  $\beta > 1$  and  $0 < \beta < 1$  are equivalent (both satisfy the boundary conditions).

Also based on the formulas (97) and (98) may be noted that in the case of  $\beta > 1$  values of speeds  $v_{x1}$  and  $v_{x2}$  of the traffic point in fixed  $O_1x_1y_1z_1$  and moving  $O_2x_2y_2z_2$  inertial reference systems may be located only in the range of  $-v_{xkp1}$  to  $+v_{xkp1}$ .

In turn, in the case  $0 < \beta < 1$ , considering the formula (113), you can see that when values  $v_{x2} > 0$  and  $V > 0$  the change of the value of speed  $v_{x2}$  of the traffic point in the moving reference system  $O_2x_2y_2z_2$  from  $0$  to  $v_{xkp2}^2/V$  lead to the changing of the value of speed  $v_{x1}$  of traffic this point in the fixed reference system  $O_1x_1y_1z_1$  from  $V$  to  $+\infty$ , but if the change the of value of speed  $v_{x2}$  of traffic of point in the moving reference system from  $v_{xkp2}^2/V$  to  $\infty$  the value of speed  $v_{x1}$  of traffic this point in the fixed reference system  $O_1x_1y_1z_1$  ranges from  $-\infty$  to  $0$  (ie in the field the values of  $v_{x2}$ , equal  $v_{xkp2}^2/V$ , there is a shift of value  $v_{x1}$  from  $+\infty$  to  $-\infty$  ).

### 3.2. The definition of the values of the transition coefficient $\beta$

With the help of the body mass dependencies (150) and (171) of the speed  $V$  we will try to establish what really is the range of the transition coefficient  $\beta$  - in

$\beta > 1$  or  $0 < \beta < 1$ , as these ranges are mutually exclusive in connection with the dependency transition coefficient  $\beta$  of speed  $V$ .

Let's try to meet this challenge, considering the law of the conservation of the momentum (the law of the conservation of the mechanical energy) in the event that all or part of the bodies (the material points), constituting the closed mechanical system, do not move linearly.

To this end, turn to the simplest example.

### 3.2.1. Example № 3

Assume that there are two inertial reference systems similar to the reference systems shown in Fig. 1 - fixed  $O_1x_1y_1z_1$  and moving  $O_2x_2y_2z_2$ , which is moving at speed  $V$  parallel to the  $O_1x_1$  axis relative to the system  $O_1x_1y_1z_1$ .

Assume that there is a closed mechanical system of bodies, shown in Fig. 5, and composed of point bodies 1 and 2, with the equal mass  $M_0$  in a state of the rest, and the thread 3.

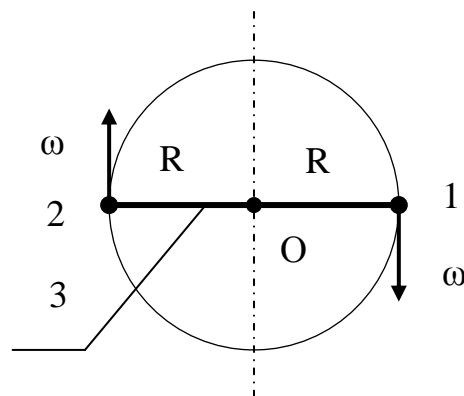


Fig. 5

Bodies 1 and 2 are connected with the absolutely rigid (not deformable) thread 3 with no mass.

Bodies 1 and 2 rotate with an angular speed  $\omega$  around the common center of mass - point  $O$ . The distance from body 1 (body 2) to the point  $O$  is  $R$ .

Put the closed mechanical system of bodies 1 and 2 with thread 3 in the moving reference system  $O_2x_2y_2z_2$  so that point  $O$  is fixed in this reference system

and coincides with the beginning of the coordinates - point  $O_2$ . The rotation of bodies 1 and 2 happen to be clockwise in the plane  $O_2x_2y_2$ , as shown in Fig. 6.

Also, let's say, at starting time ( $t_2=0$ ) in reference system  $O_2x_2y_2z_2$ , bodies 1 and 2 were on the  $O_2x_2$  axis with body 1 having positive coordinates, and body 2 - negative coordinates.

Based on the foregoing, it may be noted that in the moving reference system  $O_2x_2y_2z_2$  at any time  $t_2$  bodies 1 and 2 will have speed  $v_{21}$  and  $v_{22}$  respectively equal:

$$v_{21} = v_{22} = v = \omega \cdot R \quad (191)$$

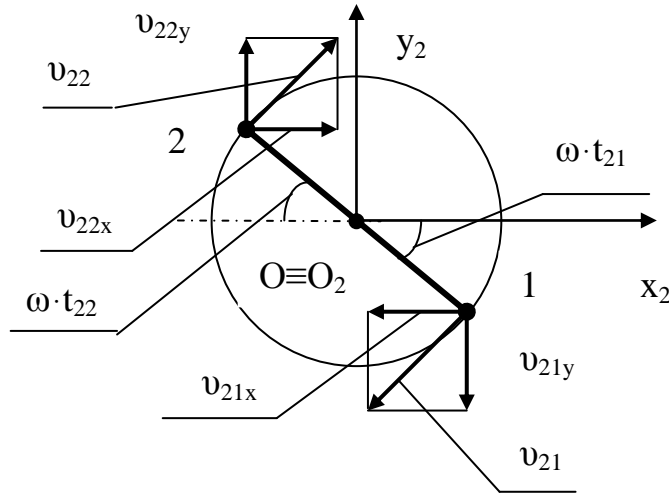


Рис. 6

While the projection  $v_{21x}$  and  $v_{21y}$  of the speed of body 1 and the projection  $v_{22x}$  and  $v_{22y}$  of the speed of body 2 on the  $O_2x_2$  and  $O_2y_2$  axes respectively for the moments of time  $t_{21}$  and  $t_{22}$ , will be equal to:

$$v_{21x} = - [v \cdot \sin(\omega \cdot t_{21})] \quad (192)$$

$$v_{21y} = - [v \cdot \cos(\omega \cdot t_{21})] \quad (193)$$

$$v_{22x} = v \cdot \sin(\omega \cdot t_{22}) \quad (194)$$

$$v_{22y} = v \cdot \cos(\omega \cdot t_{22}) \quad (195)$$

The connection between the coordinates  $x_{21}$  and  $y_{21}$  of body 1, depending on time  $t_{21}$ , and the connection between the coordinates  $x_{22}$  and  $y_{22}$  of the body 2, depending on time  $t_{22}$  in the moving reference system  $O_2x_2y_2z_2$  can be written as:

$$x_{21} = R \cdot \cos(\omega \cdot t_{21}) \quad (196)$$

$$y_{21} = - [R \cdot \sin(\omega \cdot t_{21})] \quad (197)$$

$$x_{22} = - [R \cdot \cos(\omega \cdot t_{22})] \quad (198)$$

$$y_{22} = R \cdot \sin(\omega \cdot t_{22}) \quad (199)$$

Based on equations (34) and (36), you can write the connection between the coordinates  $\mathbf{x}_{11}$  and  $\mathbf{y}_{11}$  of body 1 at time  $t_{11}$  in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and the coordinates  $\mathbf{x}_{21}$  and  $\mathbf{y}_{21}$  of body 1 at time  $t_{21}$  in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ :

$$x_{11} = \beta \cdot [x_{21} + (V \cdot t_{21})] \quad (200)$$

$$y_{11} = y_{21} \quad (201)$$

Similarly, using the equations (34) and (36), you can write the connection between coordinates  $\mathbf{x}_{12}$  and  $\mathbf{y}_{12}$  of body 2 at time  $t_{12}$  in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and the coordinates  $\mathbf{x}_{22}$  and  $\mathbf{y}_{22}$  of body 2 at time  $t_{22}$  in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ :

$$x_{12} = \beta \cdot [x_{22} + (V \cdot t_{22})] \quad (202)$$

$$y_{12} = y_{22} \quad (203)$$

Using formula (38) you can write the connection between times  $t_{11}$ ,  $t_{21}$  and  $t_{12}$ ,  $t_{22}$ :

$$t_{11} = \frac{(\beta^2 - 1) \cdot x_{21}}{\beta \cdot V} + (\beta \cdot t_{21}) \quad (204)$$

$$t_{12} = \frac{(\beta^2 - 1) \cdot x_{22}}{\beta \cdot V} + (\beta \cdot t_{22}) \quad (205)$$

In the given example, we will be interested in the position of bodies 1 and 2 in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at the same time. That is, when:

$$t_{11} = t_{12} \quad (206)$$

Then equation (206), taking into the account formulas (196), (198), (200), (202), (204) and (205) will look like this:

$$\begin{aligned} & \frac{(\beta^2 - 1) \cdot R \cdot \cos(\omega \cdot t_{21})}{\beta \cdot V} + (\beta \cdot t_{21}) = \\ & = \frac{(1 - \beta^2) \cdot R \cdot \cos(\omega \cdot t_{22})}{\beta \cdot V} + (\beta \cdot t_{22}) \end{aligned} \quad (207)$$



In the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  in fulfilling conditions (206) the position of the bodies 1 and 2 is interesting, when:

$$t_{21} = t_{22} = t_{2p} \quad (208)$$

Substituting condition (208) in equation (207) for the case, when  $(\boldsymbol{\omega} \cdot \mathbf{t}_{2p}) < \pi$ , we get:

$$\boldsymbol{\omega} \cdot \mathbf{t}_{2p} = \frac{\pi}{2} \quad (209)$$

That is to fulfill the conditions (206) and (208) in the considered times, bodies 1 and 2 must be on the line parallel to axis  $\mathbf{O}_2\mathbf{y}_2$  ( $\mathbf{O}_1\mathbf{y}_1$ ).

Also in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  in the fulfilling conditions (206) the position of the bodies 1 and 2 are interesting when:

$$t_{21} = 0 \quad (210)$$

The importance of time  $t_{22}$  in fulfilling the conditions (206) and (210) denoted  $t_{22\tau}$ , equation (207) will look like this:

$$t_{22\tau} = \left(1 - \frac{1}{\beta^2}\right) \cdot [1 + \cos(\boldsymbol{\omega} \cdot t_{22\tau})] \cdot \frac{R}{V} \quad (211)$$

or:

$$\boldsymbol{\omega} \cdot t_{22\tau} = \left(1 - \frac{1}{\beta^2}\right) \cdot [1 + \cos(\boldsymbol{\omega} \cdot t_{22\tau})] \cdot \frac{v}{V} \quad (212)$$

As can be seen from the equation (212), the value of the time  $t_{22\tau}$  depending on the transition coefficient  $\beta$  can be:

$$- \quad \mathbf{t}_{22\tau} > 0 \text{ with } \beta > 1 ; \quad (213)$$

$$- \quad \mathbf{t}_{22\tau} < 0 \text{ with } 0 < \beta < 1 ; \quad (214)$$

$$- \quad \mathbf{t}_{22\tau} = 0 \text{ with } \beta = 1 . \quad (215)$$

Now we may proceed to verify compliance with the law of conservation of momentum (of the kinetic energy conservation law).

Consider two times in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$ .

### 3.2.1.1. Time $t_{1p}$

Under the terms (206) and (208) for bodies 1 and 2, time  $t_{1p}$  in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  will correspond to time  $t_{2p}$  in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ .

As shown in Fig. 7, according to equations (209), (192)-(195) in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  at time  $t_{2p}$ , bodies 1 and 2 respectively have the following meanings for projections  $v_{21xp}$ ,  $v_{21yp}$  and  $v_{22xp}$ ,  $v_{22yp}$  of its motion on the axes  $\mathbf{O}_2\mathbf{x}_2$  and  $\mathbf{O}_2\mathbf{y}_2$ :

$$v_{21xp} = -v \quad (216)$$

$$v_{21yp} = 0 \quad (217)$$

$$v_{22xp} = v \quad (218)$$

$$v_{22yp} = 0 \quad (219)$$

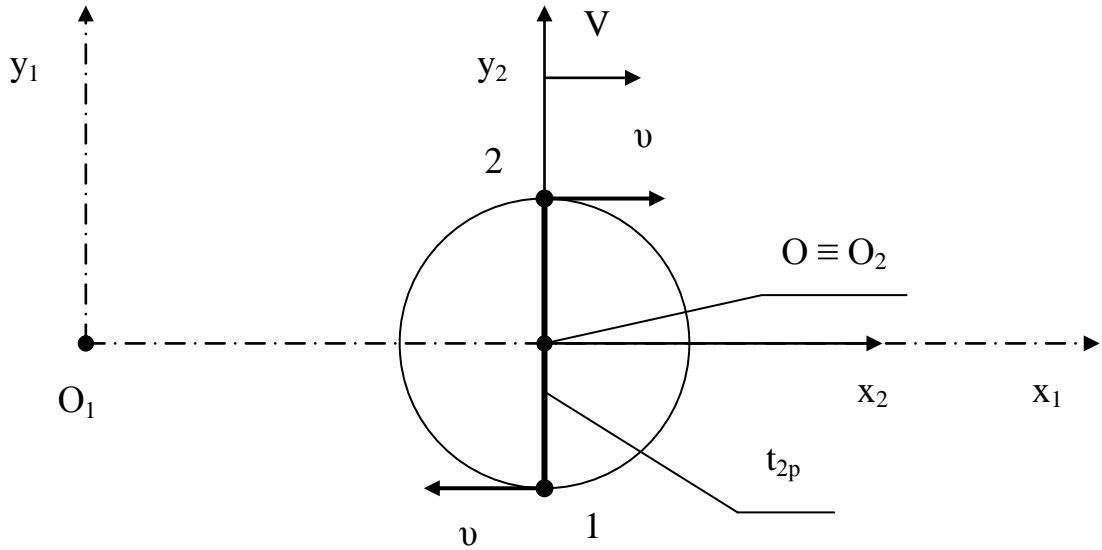


Fig. 7

Then, on the basis of formulas (40), (42) and the equality (216)-(219), in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at time  $t_{1p}$ , bodies 1 and 2 respectively will have the following meaning for projections  $v_{11xp}$ ,  $v_{11yp}$  and  $v_{12xp}$ ,  $v_{12yp}$  of its motion on the axes  $\mathbf{O}_1\mathbf{x}_1$  and  $\mathbf{O}_1\mathbf{y}_1$ :

$$v_{11xp} = \frac{V - v}{1 - \frac{(\beta^2 - 1) \cdot v}{\beta^2 \cdot V}} \quad (220)$$

$$v_{11yp} = 0 \quad (221)$$

$$v_{12xp} = \frac{V + v}{\frac{(\beta^2 - 1) \cdot v}{\beta^2 \cdot V} + 1} \quad (222)$$

$$v_{12yp} = 0 \quad (223)$$

### 3.2.1.2. Time $t_{1T}$

Under the terms (206) and (210), time  $t_{1T}$  in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  will correspond to time  $t_{21} = 0$  for body 1 and time  $t_{22T}$  for body 2 in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ .

As shown in Fig. 8, in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  at time  $t_{21} = 0$  body 1 and at time  $t_{22T}$  body 2 respectively have the following meanings for projections  $v_{21xT}$ ,  $v_{21yT}$  and  $v_{22xT}$ ,  $v_{22yT}$  of its motion on  $\mathbf{O}_2\mathbf{x}_2$  and  $\mathbf{O}_2\mathbf{y}_2$  axes with:

$$v_{21xT} = 0 \quad (224)$$

$$v_{21yT} = -v \quad (225)$$

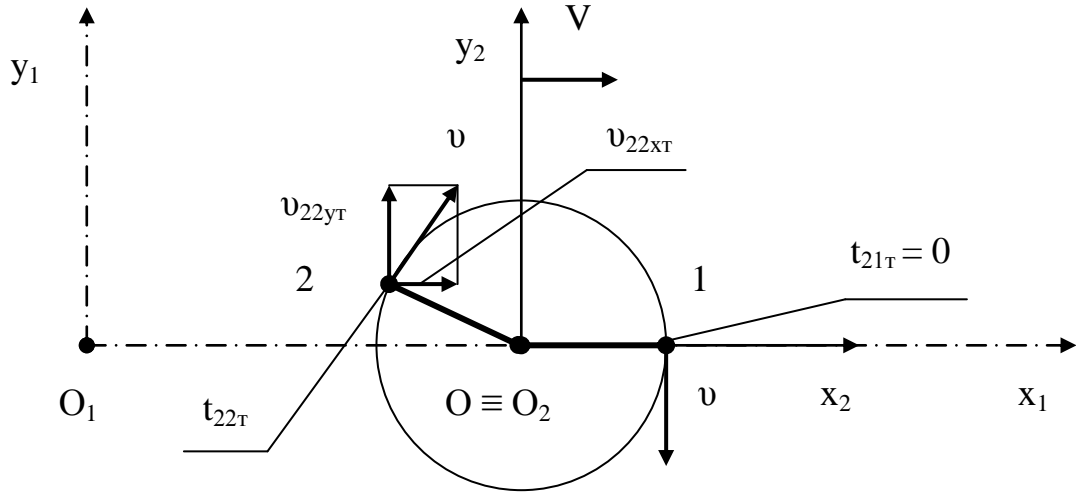


Fig. 8

Then, on the basis of formulas (40), (42) and the equality (224), (225) in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at time  $t_{1T}$  body 1 and body 2 respectively, will have the meaning for projections  $v_{11xT}$ ,  $v_{11yT}$  and  $v_{12xT}$ ,  $v_{12yT}$  of its motion on the  $\mathbf{O}_1\mathbf{x}_1$  and  $\mathbf{O}_1\mathbf{y}_1$  axes with:

$$v_{11xT} = V \quad (226)$$

$$v_{11y_T} = -\frac{v}{\beta} \quad (227)$$

$$v_{12x_T} = \frac{V + v_{22x_T}}{\frac{(\beta^2 - 1) \cdot v_{22x_T}}{\beta^2 \cdot V} + 1} \quad (228)$$

$$v_{12y_T} = \frac{v_{22y_T}}{\frac{(\beta^2 - 1) \cdot v_{22x_T}}{\beta \cdot V} + \beta} \quad (229)$$

Given condition (213), that with the transition coefficient  $\beta > 1$  and time  $t_{22_T} > 0$ , it may be noted that with the transition coefficient  $\beta > 1$  the projection  $v_{22y_T}$  of the speed will be aimed in the direction of the  $\mathbf{O}_2\mathbf{y}_2$  axis.

Also, on the basis of conditions (214), claims that with the transition coefficient  $0 < \beta < 1$ , time  $t_{22_T} < 0$ , it may be noted that with the transition coefficient  $0 < \beta < 1$ , the projection  $v_{22y_T}$  of the speed will be the direction opposite the direction of the  $\mathbf{O}_2\mathbf{y}_2$  axis.

From equations (194) and (195) may be obtained:

$$v_{22x_T}^2 + v_{22y_T}^2 = v^2 \quad (230)$$

### 3.2.1.3. The equations of the law of conservation of momentum and the law of conservation of mechanical energy for example № 3

In the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at time  $t_{1p}$  body 1 and body 2 respectively will have the following meanings for the kinetic energy  $\mathbf{E}_{k11p}$  and  $\mathbf{E}_{k12p}$  and the projections  $\mathbf{P}_{11xp}$ ,  $\mathbf{P}_{11yp}$  and  $\mathbf{P}_{12xp}$ ,  $\mathbf{P}_{12yp}$  of the momentum on axes  $\mathbf{O}_1\mathbf{x}_1$  and  $\mathbf{O}_1\mathbf{y}_1$ :

$$P_{11xp} = M_0 \cdot f(V = v_{11xp}) \cdot v_{11xp} \quad (231)$$

$$P_{12xp} = M_0 \cdot f(V = v_{12xp}) \cdot v_{12xp} \quad (232)$$

$$P_{11yp} = 0 \quad (233)$$

$$P_{12yp} = 0 \quad (234)$$

$$E_{k11p} = M_0 \cdot \int_0^{v_{11xp}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \quad (235)$$

$$E_{k12p} = M_0 \cdot \int_0^{v_{12xp}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \quad (236)$$

In the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at time  $t_{1T}$  body 1 and body 2 respectively, will have the following meanings for the kinetic energy  $E_{k11T}$  and  $E_{k12T}$  and the projections  $P_{11xT}$ ,  $P_{11yT}$  and  $P_{12xT}$ ,  $P_{12yT}$  of momentum on the  $\mathbf{O}_1\mathbf{x}_1$  and  $\mathbf{O}_1\mathbf{y}_1$  axes:

$$P_{11xT} = M_0 \cdot f \left[ V = \sqrt{v_{11xT}^2 + v_{11yT}^2} \right] \cdot v_{11xT} \quad (237)$$

$$P_{12xT} = M_0 \cdot f \left[ V = \sqrt{v_{12xT}^2 + v_{12yT}^2} \right] \cdot v_{12xT} \quad (238)$$

$$P_{11yT} = M_0 \cdot f \left[ V = \sqrt{v_{11xT}^2 + v_{11yT}^2} \right] \cdot v_{11yT} \quad (239)$$

$$P_{12yT} = M_0 \cdot f \left[ V = \sqrt{v_{12xT}^2 + v_{12yT}^2} \right] \cdot v_{12yT} \quad (240)$$

$$E_{k11T} = M_0 \cdot \int_0^{\sqrt{v_{11xT}^2 + v_{11yT}^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \quad (241)$$

$$E_{k12T} = M_0 \cdot \int_0^{\sqrt{v_{12xT}^2 + v_{12yT}^2}} \{[f(V) \cdot V] + [f'(V) \cdot V^2]\} \cdot dV \quad (242)$$

Due to the fact that the mechanical system of bodies 1 and 2 (and the thread 3) is closed, the law of the conservation of the momentum allows writing for times  $t_{1p}$  and  $t_{1T}$ , the following equation:

$$P_{11xp} + P_{12xp} = P_{11xT} + P_{12xT}$$

or

$$\begin{aligned} & \{M_0 \cdot f(V = v_{11xp}) \cdot v_{11xp}\} + \{M_0 \cdot f(V = v_{12xp}) \cdot v_{12xp}\} = \\ & = \left\{ M_0 \cdot f \left[ V = \sqrt{v_{11xT}^2 + v_{11yT}^2} \right] \cdot v_{11xT} \right\} + \\ & + \left\{ M_0 \cdot f \left[ V = \sqrt{v_{12xT}^2 + v_{12yT}^2} \right] \cdot v_{12xT} \right\} \end{aligned} \quad (243)$$

$$P_{11yp} + P_{12yp} = P_{11yT} + P_{12yT}$$

or

$$0 = \left\{ M_0 \cdot f \left[ V = \sqrt{v_{11xT}^2 + v_{11yT}^2} \right] \cdot v_{11yT} \right\} + \\ + \left\{ M_0 \cdot f \left[ V = \sqrt{v_{12xT}^2 + v_{12yT}^2} \right] \cdot v_{12yT} \right\} \quad (244)$$

Also due to the fact that the mechanical system of bodies 1 and 2 (and thread 3) is closed and the potential energy of the bodies 1 and 2 do not change, the law of the conservation of the mechanical energy allows writing for times  $t_{1p}$  and  $t_{1T}$ , the following equation:

$$E_{K11p} + E_{K12p} = E_{K11T} + E_{K12T}$$

or

$$\langle M_0 \cdot \int_0^{v_{11xp}} \{ [f(V) \cdot V] + [f'(V) \cdot V^2] \} \cdot dV \rangle + \\ + \langle M_0 \cdot \int_0^{v_{12xp}} \{ [f(V) \cdot V] + [f'(V) \cdot V^2] \} \cdot dV \rangle = \\ = \langle M_0 \cdot \int_0^{\sqrt{v_{11xT}^2 + v_{11yT}^2}} \{ [f(V) \cdot V] + [f'(V) \cdot V^2] \} \cdot dV \rangle + \\ + \langle M_0 \cdot \int_0^{\sqrt{v_{12xT}^2 + v_{12yT}^2}} \{ [f(V) \cdot V] + [f'(V) \cdot V^2] \} \cdot dV \rangle \quad (245)$$

#### **3.2.1.4. The determination of the environment in which the law of conservation of momentum allows us to maintain example № 3 with the transition coefficient $\beta \geq 1$**

In the event that the transition coefficient  $\beta \geq 1$ , the values of the transition coefficient  $\beta$  and the function  $f(V)$  is determined by:

$$\beta_{>}^2 = \frac{1}{1 - \frac{V^2}{v_{xkp1}^2}} \quad (59)$$

$$f(V)_> = \frac{1}{\sqrt{1 - \frac{V^2}{v_{\text{кр}1}^2}}} \quad (149)$$

Then, taking into the account formula (149), equations (243) and (244) will look like this:

$$\frac{M_0 \cdot v_{11xp}}{\sqrt{1 - \frac{v_{11xp}^2}{v_{\text{кр}1}^2}}} + \frac{M_0 \cdot v_{12xp}}{\sqrt{1 - \frac{v_{12xp}^2}{v_{\text{кр}1}^2}}} = \frac{M_0 \cdot v_{11xT}}{\sqrt{1 - \frac{v_{11xT}^2 + v_{11yT}^2}{v_{\text{кр}1}^2}}} + \frac{M_0 \cdot v_{12xT}}{\sqrt{1 - \frac{v_{12xT}^2 + v_{12yT}^2}{v_{\text{кр}1}^2}}} \quad (246)$$

$$0 = \frac{M_0 \cdot v_{11yT}}{\sqrt{1 - \frac{v_{11xT}^2 + v_{11yT}^2}{v_{\text{кр}1}^2}}} + \frac{M_0 \cdot v_{12yT}}{\sqrt{1 - \frac{v_{12xT}^2 + v_{12yT}^2}{v_{\text{кр}1}^2}}} \quad (247)$$

Formulas (220)-(223) and (226)-(229), taking into the account formula (59) can be written:

$$v_{11xp} = \frac{V - v}{1 - \frac{V \cdot v}{v_{\text{кр}1}^2}} \quad (248)$$

$$v_{12xp} = \frac{V + v}{1 + \frac{V \cdot v}{v_{\text{кр}1}^2}} \quad (249)$$

$$v_{11xT} = V \quad (226)$$

$$v_{11yT} = - \left( v \cdot \sqrt{1 - \frac{V^2}{v_{\text{кр}1}^2}} \right) \quad (250)$$

$$v_{12xT} = \frac{V + v_{22xT}}{1 + \frac{V \cdot v_{22xT}}{v_{\text{кр}1}^2}} \quad (251)$$

$$v_{12yT} = \frac{v_{22yT} \cdot \sqrt{1 - \frac{V^2}{v_{\text{кр}1}^2}}}{1 + \frac{V \cdot v_{22xT}}{v_{\text{кр}1}^2}} \quad (252)$$

By placing the speed projections  $\mathbf{v}_{11xp}$ ,  $\mathbf{v}_{12xp}$ ,  $\mathbf{v}_{11xt}$ ,  $\mathbf{v}_{11yt}$ ,  $\mathbf{v}_{12xt}$  and  $\mathbf{v}_{12yt}$  of formulas (226), (248)-(252) in equations (246) and (247), and using formula (230), we get:

$$\begin{aligned} & \frac{M_0 \cdot (V - v)}{\sqrt{1 - \frac{v^2}{v_{xkp1}^2}} \cdot \sqrt{1 - \frac{V^2}{v_{xkp1}^2}}} + \frac{M_0 \cdot (V + v)}{\sqrt{1 - \frac{v^2}{v_{xkp1}^2}} \cdot \sqrt{1 - \frac{V^2}{v_{xkp1}^2}}} = \\ & = \frac{M_0 \cdot V}{\sqrt{1 - \frac{v^2}{v_{xkp1}^2}} \cdot \sqrt{1 - \frac{V^2}{v_{xkp1}^2}}} + \frac{M_0 \cdot (V + v_{22xt})}{\sqrt{1 - \frac{v^2}{v_{xkp1}^2}} \cdot \sqrt{1 - \frac{V^2}{v_{xkp1}^2}}} \end{aligned} \quad (253)$$

$$0 = - \frac{M_0 \cdot v}{\sqrt{1 - \frac{v^2}{v_{xkp1}^2}}} + \frac{M_0 \cdot v_{22yt}}{\sqrt{1 - \frac{v^2}{v_{xkp1}^2}}} \quad (254)$$

or:

$$\begin{aligned} V - v + V + v &= V + V + v_{22xt} \\ 0 &= -v + v_{22yt} \end{aligned}$$

From equations (253) and (254) we receive the necessary conditions (the values of the projections speeds  $\mathbf{v}_{22xt}$  and  $\mathbf{v}_{22yt}$ ), by which in example № 3 with the transition coefficient  $\beta \geq 1$ , the law of the conservation of the momentum will be implemented in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$ :

$$v_{22xt} = 0 \quad (255)$$

$$v_{22yt} = v \quad (256)$$

Equations (255) and (256) show that the value of the speed projections  $\mathbf{v}_{22xt}$  and  $\mathbf{v}_{22yt}$  is not dependent on speed  $\mathbf{V}$  (and the hence not dependent on the transition coefficient  $\beta$ ).

Substituting conditions (255) and (256) in equations (194) and (195), we get:

$$t_{22\tau} = t_{21\tau} = 0 \quad (257)$$

And substituting equation (257) in the formula (212):

$$\omega \cdot 0 = \left(1 - \frac{1}{\beta^2}\right) \cdot [1 + 1] \cdot \frac{v}{V} \quad (258)$$



we have another condition in implementation of the law of the conservation of momentum in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  for example № 3:

$$\beta = 1 \quad (259)$$

Thus, it can be concluded, that in the closed mechanical system of the bodies, considered in the example № 3, with the values of the transition coefficient  $\beta > 1$  the law of conservation of momentum is not met.

### 3.2.1.5. The determination of the environment in which the conservation of mechanical energy is maintained for example № 3 with the transition coefficient $\beta \geq 1$

In addition to the implementation of the law of conservation of momentum we will try to retain the implementation of the law of conservation of mechanical energy.

Given formula (149) equation (245) will look like this:

$$\begin{aligned} & \left[ M_0 \cdot v_{xkp1}^2 \cdot \left( \frac{1}{\sqrt{1 - \frac{v_{11xp}^2}{v_{xkp1}^2}}} - 1 \right) \right] + \left[ M_0 \cdot v_{xkp1}^2 \cdot \left( \frac{1}{\sqrt{1 - \frac{v_{12xp}^2}{v_{xkp1}^2}}} - 1 \right) \right] = \\ & = \left[ M_0 \cdot v_{xkp1}^2 \cdot \left( \frac{1}{\sqrt{1 - \frac{v_{11xT}^2 + v_{11yT}^2}{v_{xkp1}^2}}} - 1 \right) \right] + \\ & + \left[ M_0 \cdot v_{xkp1}^2 \cdot \left( \frac{1}{\sqrt{1 - \frac{v_{12xT}^2 + v_{12yT}^2}{v_{xkp1}^2}}} - 1 \right) \right] \quad (260) \end{aligned}$$

By placing the speed projections  $v_{11xp}$ ,  $v_{12xp}$ ,  $v_{11xT}$ ,  $v_{11yT}$ ,  $v_{12xT}$  and  $v_{12yT}$  of the formulas (226), (248)-(252) in equation (260), taking into account formula (230) we receive:

$$\begin{aligned}
& \frac{\left(1 - \frac{v \cdot V}{V_{\text{xkp1}}^2}\right) \cdot V_{\text{xkp1}}}{\sqrt{\left(1 - \frac{V^2}{V_{\text{xkp1}}^2}\right) \cdot (V_{\text{xkp1}}^2 - v^2)}} + \frac{\left(1 + \frac{v \cdot V}{V_{\text{xkp1}}^2}\right) \cdot V_{\text{xkp1}}}{\sqrt{\left(1 - \frac{V^2}{V_{\text{xkp1}}^2}\right) \cdot (V_{\text{xkp1}}^2 - v^2)}} = \\
& = \frac{V_{\text{xkp1}}}{\sqrt{\left(1 - \frac{V^2}{V_{\text{xkp1}}^2}\right) \cdot (V_{\text{xkp1}}^2 - v^2)}} + \frac{\left(1 + \frac{v_{22\text{xT}} \cdot V}{V_{\text{xkp1}}^2}\right) \cdot V_{\text{xkp1}}}{\sqrt{\left(1 - \frac{V^2}{V_{\text{xkp1}}^2}\right) \cdot (V_{\text{xkp1}}^2 - v^2)}} \quad (261)
\end{aligned}$$

Or:

$$1 - \frac{v \cdot V}{V_{\text{xkp1}}^2} + 1 + \frac{v \cdot V}{V_{\text{xkp1}}^2} = 1 + 1 + \frac{v_{22\text{xT}} \cdot V}{V_{\text{xkp1}}^2}$$

From equation (261) we receive the necessary condition (the values of the speed projections  $v_{22\text{xT}}$  and  $v_{22\text{yT}}$ ), which in example № 3 with the transition coefficient  $\beta \geq 1$  in the fixed inertial reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$ , implemented the law of the conservation of the mechanical energy will be implemented, assuming that the potential energy system of bodies 1 and 2 do not change:

$$v_{22\text{xT}} = 0 \quad (255)$$

Then, based on formula (230), we get:

$$v_{22\text{yT}} = v \quad (256)$$

This leads to the conclusion: the condition for the performance of the law of conservation of mechanical energy (as the condition for the implementation of the law of the conservation of momentum) in the fixed inertial reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  for example № 3 is:

$$\beta = 1 \quad (259)$$

Thus, it turns out that in the closed mechanical system of the bodies, considered in example № 3, with the values of the transition coefficient  $\beta > 1$ , the law of conservation of mechanical energy is not met.

Similarly, it can be shown that in the closed mechanical system of the bodies, considered in the example № 3, with the values of the transition coefficient  $\beta > 1$  the law of conservation of angular momentum will not be implemented.

The numerical calculations confirm the foregoing.

### 3.2.1.6. The numerical calculation for example № 3 with the transition coefficient $\beta > 1$

Assume that:  $V / v_{xkp1} = 0,9$ ,  $v / v_{xkp1} = 0,6$ .

The equation (212), taking into the account formula (59) can be written as:

$$\omega \cdot t_{22T} = \frac{v \cdot V \cdot [1 + \cos(\omega \cdot t_{22T})]}{v_{xkp1}^2} \quad (262)$$

Then get:

$\omega \cdot t_{22T} = 0,8828669738 = 0.8828669738$ , the projections  $v_{22xT} / v_{xkp1} = 0,4635374427$  and  $v_{22yT} / v_{xkp1} = 0,3809633042$  of the speed of body 2 in the moving reference system  $O_2x_2y_2z_2$ .

In the fixed inertial reference system  $O_1x_1y_1z_1$ :

a) at the time  $t_{1p}$ :

Period of time	Object	Name value	Value
$t_{1p}$	Body 1	projection of the momentum on axis $O_1x_1$ $K_{11xp} / (M_0 \cdot v_{xkp1})$	0,860309002
		kinetic energy $E_{K11p} / v_{xkp1}^2$	0,31914047
	Body 2	projection of the momentum on axis $O_1x_1$ $K_{12xp} / (M_0 \cdot v_{xkp1})$	4,30154501
		kinetic energy $E_{K12p} / v_{xkp1}^2$	3,416252877
	System of bodies 1 and 2	projection of the momentum on axis $O_1x_1$ $K_{12x\Sigma p} / (M_0 \cdot v_{xkp1})$	5,161854012
		projection of the momentum on axis $O_1y_1$ $K_{12y\Sigma p} / (M_0 \cdot v_{xkp1})$	0
		kinetic energy $E_{Kp} / (M_0 \cdot v_{xkp1}^2)$	3,735393347

b) at time  $t_{1T}$  :

Period of time	Object	Name value	Value
$t_{1T}$	Body 1	projection of the momentum on axis $O_1 x_1$ $K_{11xT} / (M_0 \cdot v_{xkp1})$	2,580927006
		projection of the momentum on axis $O_1 y_1$ $K_{11yT} / (M_0 \cdot v_{xkp1})$	- 0,75
		kinetic energy $E_{K11T} / v_{xkp1}^2$	1,092373316
	Body 2	projection of the momentum on axis $O_1 x_1$ $K_{12xT} / (M_0 \cdot v_{xkp1})$	3,9102117884
		projection of the momentum on axis $O_1 y_1$ $K_{12yT} / (M_0 \cdot v_{xkp1})$	0,4762041303
		kinetic energy $E_{K12T} / v_{xkp1}^2$	3,064052977
	System of bodies 1 and 2	projection of the momentum on axis $O_1 x_1$ $K_{12x\Sigma T} / (M_0 \cdot v_{xkp1})$	6,491138794
		projection of the momentum on axis $O_1 y_1$ $K_{12y\Sigma T} / (M_0 \cdot v_{xkp1})$	- 0,2737958696
		kinetic energy $E_{KT} / (M_0 \cdot v_{xkp1}^2)$	4,931749651

The law of conservation of momentum is not met because:  
5,161854012 # 6,491138794 and - 0,2737958696 # 0.

The law of the conservation of the mechanical energy is not met because:  
3,735393347 # 4,931749651.

**3.2.1.7. The determination of the environment in which the law  
of conservation of momentum is required to maintain example № 3  
with transition coefficient  $0 < \beta \leq 1$**

In the event of transition coefficient  $0 < \beta \leq 1$ , the values of the transition coefficient  $\beta$  and the function  $f(V)$  is determined by:

$$\beta_{<}^2 = \frac{1}{1 + \frac{V^2}{v_{\text{кр}2}^2}} \quad (60)$$

$$f(V)_{<} = \frac{1}{\sqrt{1 + \frac{V^2}{v_{\text{кр}2}^2}}} \quad (170)$$

Then, taking into the account formula (170), equations (243) and (244) will look like this:

$$\frac{M_0 \cdot v_{11xp}}{\sqrt{1 + \frac{v_{11xp}^2}{v_{\text{кр}2}^2}}} + \frac{M_0 \cdot v_{12xp}}{\sqrt{1 + \frac{v_{12xp}^2}{v_{\text{кр}2}^2}}} = \frac{M_0 \cdot v_{11xT}}{\sqrt{1 + \frac{v_{11xT}^2 + v_{11yT}^2}{v_{\text{кр}2}^2}}} + \frac{M_0 \cdot v_{12xT}}{\sqrt{1 + \frac{v_{12xT}^2 + v_{12yT}^2}{v_{\text{кр}2}^2}}} \quad (263)$$

$$0 = \frac{M_0 \cdot v_{11yT}}{\sqrt{1 + \frac{v_{11xT}^2 + v_{11yT}^2}{v_{\text{кр}2}^2}}} + \frac{M_0 \cdot v_{12yT}}{\sqrt{1 + \frac{v_{12xT}^2 + v_{12yT}^2}{v_{\text{кр}2}^2}}} \quad (264)$$

Formulas (220)-(223) and (226)-(229), taking into the account formula (60), can be written:

$$v_{11xp} = \frac{V - v}{1 + \frac{V \cdot v}{v_{\text{кр}2}^2}} \quad (265)$$

$$v_{12xp} = \frac{V + v}{1 - \frac{V \cdot v}{v_{\text{кр}2}^2}} \quad (266)$$

$$v_{11xT} = V \quad (226)$$

$$v_{11yT} = - \left( v \cdot \sqrt{1 + \frac{V^2}{v_{\text{кр}2}^2}} \right) \quad (267)$$

$$v_{12xT} = \frac{V + v_{22xT}}{1 - \frac{V \cdot v_{22xT}}{v_{xkp2}^2}} \quad (268)$$

$$v_{12yT} = \frac{v_{22yT} \cdot \sqrt{1 + \frac{V^2}{v_{xkp2}^2}}}{1 - \frac{V \cdot v_{22xT}}{v_{xkp2}^2}} \quad (269)$$

By placing the speed projections  $v_{11xp}$ ,  $v_{12xp}$ ,  $v_{11xT}$ ,  $v_{11yT}$ ,  $v_{12xT}$  and  $v_{12yT}$  of formulas (226), (265)-(269) in the equations (263) and (264), and using formula (230), we get:

$$\begin{aligned} & \frac{M_0 \cdot (V - v)}{\sqrt{1 + \frac{v^2}{v_{xkp2}^2}} \cdot \sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} + \frac{M_0 \cdot (V + v)}{\sqrt{1 + \frac{v^2}{v_{xkp2}^2}} \cdot \sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} = \\ & = \frac{M_0 \cdot V}{\sqrt{1 + \frac{v^2}{v_{xkp2}^2}} \cdot \sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} + \frac{M_0 \cdot (V + v_{22xT})}{\sqrt{1 + \frac{v^2}{v_{xkp2}^2}} \cdot \sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \end{aligned} \quad (270)$$

$$0 = - \frac{M_0 \cdot v}{\sqrt{1 + \frac{v^2}{v_{xkp2}^2}}} + \frac{M_0 \cdot v_{22yT}}{\sqrt{1 + \frac{v^2}{v_{xkp2}^2}}} \quad (271)$$

or:

$$V - v + V + v = V + V + v_{22xT}$$

$$0 = -v + v_{22yT}$$

From equations (270) and (271) we receive the necessary conditions (the values of the projections speeds  $v_{22xT}$  and  $v_{22yT}$ ), in which, in example № 3, with the transition coefficient  $0 < \beta \leq 1$  the law of conservation of momentum will be implemented in the fixed reference system  $O_1x_1y_1z_1$ :

$$v_{22xT} = 0 \quad (255)$$

$$v_{22yT} = v \quad (256)$$

Equations (255) and (256) show that the speed projections  $v_{22xT}$  and  $v_{22yT}$  are not dependent on speed  $V$  (and the hence not dependent on the value of the transition coefficient  $\beta$ ).

Substituting conditions (255) and (256) in equations (194) and (195), we get:

$$t_{22T} = t_{21T} = 0 \quad (257)$$

And substituting equation (257) in formula (212):

$$\omega \cdot 0 = \left(1 - \frac{1}{\beta^2}\right) \cdot [1 + 1] \cdot \frac{v}{V} \quad (258)$$

we have another condition for implementation of the law of conservation of momentum in the fixed reference system  $O_1x_1y_1z_1$  for example № 3:

$$\beta = 1 \quad (259)$$

Thus, it can be concluded that in the closed mechanical system of the bodies, considered in example № 3, with the values of the transition coefficient  $0 < \beta < 1$ , the law of conservation of momentum is not met.

### **3.2.1.8. The determination of the environment in which the law of conservation of mechanical energy is maintained for example № 3 with the transition coefficient $0 < \beta < 1$**

In addition to the conditions of the implementation of the law of conservation of momentum we will try to keep the conditions of implementation of the law of conservation of mechanical energy.

Given formula (170), equation (245) will look like this:

$$\begin{aligned} & \left[ M_0 \cdot v_{xkp2}^2 \cdot \left( 1 - \frac{1}{\sqrt{1 + \frac{v_{11xp}^2}{v_{xkp2}^2}}} \right) \right] + \left[ M_0 \cdot v_{xkp2}^2 \cdot \left( 1 - \frac{1}{\sqrt{1 + \frac{v_{12xp}^2}{v_{xkp2}^2}}} \right) \right] = \\ & = \left[ M_0 \cdot v_{xkp2}^2 \cdot \left( 1 - \frac{1}{\sqrt{1 + \frac{v_{11xT}^2 + v_{11yT}^2}{v_{xkp2}^2}}} \right) \right] + \end{aligned}$$

$$+ \left[ M_0 \cdot v_{\text{xkp2}}^2 \cdot \left( 1 - \frac{1}{\sqrt{1 + \frac{v_{12\text{xT}}^2 + v_{12\text{yT}}^2}{v_{\text{xkp2}}^2}}} \right) \right] \quad (272)$$

By placing the speed projections  $v_{11\text{xP}}$ ,  $v_{12\text{xP}}$ ,  $v_{11\text{xT}}$ ,  $v_{11\text{yT}}$ ,  $v_{12\text{xT}}$  and  $v_{12\text{yT}}$  of formulas (226), (265)-(269) in equation (272), taking into account formula (230), we get:

$$\begin{aligned} & \frac{\left(1 + \frac{v \cdot V}{v_{\text{xkp2}}^2}\right) \cdot v_{\text{xkp2}}}{\sqrt{\left(1 + \frac{V^2}{v_{\text{xkp2}}^2}\right) \cdot (v_{\text{xkp2}}^2 + v^2)}} + \frac{\left(1 - \frac{v \cdot V}{v_{\text{xkp1}}^2}\right) \cdot v_{\text{xkp2}}}{\sqrt{\left(1 + \frac{V^2}{v_{\text{xkp2}}^2}\right) \cdot (v_{\text{xkp2}}^2 + v^2)}} = \\ & = \frac{v_{\text{xkp2}}}{\sqrt{\left(1 + \frac{V^2}{v_{\text{xkp2}}^2}\right) \cdot (v_{\text{xkp2}}^2 + v^2)}} + \frac{\left(1 - \frac{v_{22\text{xT}} \cdot V}{v_{\text{xkp2}}^2}\right) \cdot v_{\text{xkp2}}}{\sqrt{\left(1 + \frac{V^2}{v_{\text{xkp2}}^2}\right) \cdot (v_{\text{xkp2}}^2 + v^2)}} \quad (273) \end{aligned}$$

Or:

$$1 + \frac{v \cdot V}{v_{\text{xkp2}}^2} + 1 - \frac{v \cdot V}{v_{\text{xkp2}}^2} = 1 + 1 - \frac{v_{22\text{xT}} \cdot V}{v_{\text{xkp2}}^2}$$

From equation (273) we receive the necessary condition (the value of the projection speed  $v_{22\text{xT}}$ ), in which in example № 3, with the transition coefficient  $0 < \beta < 1$  in the fixed inertial reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  will be implemented the law of conservation of mechanical energy, assuming that the potential energy system of bodies 1 and 2 does not change:

$$v_{22\text{xT}} = 0 \quad (255)$$

Then, based on formula (230), we get:

$$v_{22\text{yT}} = v \quad (256)$$

This leads to the conclusion: the conditions for the performance of the law of conservation of mechanical energy (as the condition for implementation of the law of conservation of momentum) in the fixed inertial reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  for example № 3 is:



$$\beta = 1 \quad (259)$$

Thus, it turns out that in the closed mechanical system of the bodies considered in the example № 3, with the values of the transition coefficient  $0 < \beta < 1$ , the law of conservation of mechanical energy is not met.

Similarly, it can be shown that in the closed mechanical system of the bodies, considered in example № 3, with the values of the transition coefficient  $0 < \beta < 1$  the law of conservation of angular momentum will not be implemented.

The numerical calculations confirm the foregoing.

### 3.2.1.9. The numerical calculation for example № 3 with the transition coefficient $0 < \beta < 1$

Suppose that:  $V / v_{\text{xkp2}} = 0,9$ ,  $v / v_{\text{xkp2}} = 0,6$ .

Equation (212), taking into the account formula (60), can be written as:

$$\omega \cdot t_{22T} = - \frac{v \cdot V \cdot [1 + \cos(\omega \cdot t_{22T})]}{v_{\text{xkp2}}^2} \quad (274)$$

Then get:

$\omega \cdot t_{22T} = - 0,8828669738$ , the projections  $v_{22xT} / v_{\text{xkp2}} = - 0,4635374427$  and  $v_{22yT} / v_{\text{xkp2}} = 0,3809633042$  of the speed of body 2 in the moving reference system  $O_2x_2y_2z_2$ .

In the fixed inertial reference system  $O_1x_1y_1z_1$ :

a) at time  $t_{1p}$ :

Period of time	Object	Name value	Value
$t_{1p}$	Body 1	projection of the momentum on axis $O_1x_1$ $K_{11xp} / (M_o \cdot v_{xkp2})$	0,1912108416
		kinetic energy $E_{K11p} / v_{xkp2}^2$	0,018451013
	Body 2	projection of the momentum on axis $O_1x_1$ $K_{12xp} / (M_o \cdot v_{xkp2})$	0,9560542082
		kinetic energy $E_{K12p} / v_{xkp2}^2$	0,706810043
	System of bodies 1 and 2	projection of the momentum on axis $O_1x_1$ $K_{12x\Sigma p} / (M_o \cdot v_{xkp2})$	1,1472650498
		projection of the momentum on axis $O_1y_1$ $K_{12y\Sigma p} / (M_o \cdot v_{xkp2})$	0
		kinetic energy $E_{Kp} / (M_o \cdot v_{xkp2}^2)$	0,725261056

b) at time  $t_{1T}$ :

Period of time	Object	Name value	Value
$t_{1T}$	Body 1	projection of the momentum on axis $O_1 x_1$ $K_{11xT} / (M_0 \cdot v_{xkp2})$	0,5736325249
		projection of the momentum on axis $O_1 y_1$ $K_{11yT} / (M_0 \cdot v_{xkp2})$	- 0,5144957554
		kinetic energy $E_{K11T} / v_{xkp2}^2$	0,362630528
	Body 2	projection of the momentum on axis $O_1 x_1$ $K_{12xT} / (M_0 \cdot v_{xkp2})$	0,2781879097
		projection of the momentum on axis $O_1 y_1$ $K_{12yT} / (M_0 \cdot v_{xkp2})$	0,3266733383
		kinetic energy $E_{K12T} / v_{xkp2}^2$	0,628530682
	System of bodies 1 and 2	projection of the momentum on axis $O_1 x_1$ $K_{12x\Sigma T} / (M_0 \cdot v_{xkp2})$	0,8518204346
		projection of the momentum on axis $O_1 y_1$ $K_{12y\Sigma T} / (M_0 \cdot v_{xkp2})$	- 0,187822417
		kinetic energy $E_{KT} / (M_0 \cdot v_{xkp2}^2)$	0,991161209

Conservation of momentum is not met because: 1,1472650498 # 0,8518204346 and - 0,187822417 # 0.

Conservation of mechanical energy is not met because: 0,725261056 # 0,991161209.

By the results obtained when considering example № 3, leading to consideration of the system of bodies shown in Fig. 9, in which bodies 1 and 2, described in the example № 3, retain no rigid thread, a force of attraction of body 3 (the point body), which will be the center  $O$ .

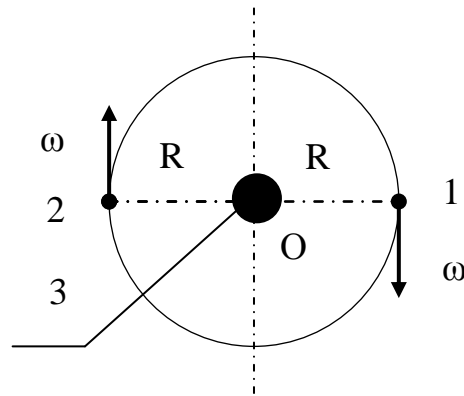


Fig. 9

### 3.2.1.10. The conclusions

As the result of the example № 3 it has been shown that when the values of the transition coefficient  $\beta$  in the ranges  $\beta > 1$  and  $0 < \beta < 1$  in the fixed reference system  $O_1x_1y_1z_1$ :

- the momentum of the closed mechanical system of bodies 1 and 2 (and the thread 3) at the time when the bodies are on the line parallel to axis  $O_1y_1$ , is not equal to the momentum of this system of bodies 1 and 2 (and the thread 3) at any other time, when bodies 1 and 2 are not on the line parallel to axis  $O_1y_1$ . That is, **in the fixed reference system  $O_1x_1y_1z_1$ , the closed mechanical system of bodies 1 and 2 (and the thread 3) will have the momentum changing over time. That is, the violation of the law of conservation of momentum for the closed mechanical system of bodies;**

- the kinetic energy (when the potential energy of the closed mechanical system of the bodies 1 and 2 are a constant value) of the closed mechanical system of bodies 1 and 2 (and the thread 3) at the time when the bodies are on the line parallel to axis  $O_1y_1$ , is not equal to the kinetic energy of this system of bodies 1 and 2 (and the thread 3) at any other time when the bodies are not on the line parallel to the axis  $O_1y_1$ . That is, **in the fixed reference system  $O_1x_1y_1z_1$  the closed**

**mechanical system of bodies 1 and 2 (and the thread 3) will have the kinetic energy changing over time, that in the constancy of the potential energy of the system, is a violation of the law of conservation of mechanical energy for the closed mechanical system .**

Similarly, it can be shown when considering the example № 3, when the values of the transition coefficient  $\beta$  in the ranges  $\beta > 1$  and  $0 < \beta < 1$  in the fixed reference system  $O_1x_1y_1z_1$ , the angular momentum of the closed mechanical system of bodies 1 and 2 (and the thread 3) at the time when the bodies are on the line parallel to the axis  $O_1y_1$ , is not equal to the angular momentum of this system at any other time, when the bodies 1 and 2 are not on the line parallel to axis  $O_1y_1$ . That is, **in the fixed reference system  $O_1x_1y_1z_1$  the closed mechanical system of e bodies 1 and 2 (and the thread 3) will have the angular momentum changing over time in violation of the law of conservation of angular momentum for the closed mechanical system of the bodies.**

The changing in time of the values of momentum, kinetic energy (when the potential energy of the closed mechanical system of the bodies 1 and 2 (and the thread 3) are a constant value), the angular momentum of the closed mechanical system of the bodies 1 and 2 (and the thread 3) in the example № 3 shows that, when the values of the transition coefficient  $\beta$  in the ranges  $\beta > 1$  and  $0 < \beta < 1$ , there is the violation of the law of the conservation of the momentum, the kinetic energy, and angular momentum.

Assuming that, the laws of conservation of momentum, the kinetic energy (with the potential energy of the system unchanged), the angular momentum of the closed mechanical system is associated with the symmetry of space and time (the homogeneity and isotropy of space and the homogeneity of time), it may be noted, that, when the values of the transition coefficient  $\beta$  in the ranges  $\beta > 1$  and  $0 < \beta < 1$ , there is a violation of the condition of symmetry of space and time.

If by definition (the original assumption) the symmetry of space and time are the area in which to apply the special theory of relativity, then when it is used in example № 3, with the violation of the condition of the symmetry of space and time when the values of the transition coefficient  $\beta$  in the ranges  $\beta > 1$  and  $0 < \beta < 1$ ,

then we have the case where there is a theory, but there is no the field for its application.

That is, in the case of the symmetry of space and time, the relationship between the coordinates and time in the inertial reference systems can be recorded using the special theory of the relativity, when the values of the transition coefficient  $\beta$  in the ranges  $\beta > 1$  and  $0 < \beta < 1$ . In short, when the symmetry of space and time is used for the inertial reference systems, the special theory of the relativity (with the transition coefficient  $\beta \neq 1$ ) does not apply.

As shown in example № 3, (regarding the conservation of momentum, mechanical energy (with the potential energy of the system unchanged) and the angular momentum of the closed mechanical system,) the condition of symmetry of space and time are carried out only with the transition coefficient  $\beta = 1$  (when  $v_{xkp1} = \infty$  or  $v_{xkp2} = \infty$ ), or when the transition coefficient  $\beta$  is not a function of the speed  $V$  of the movement of the inertial reference system.

That leads to the conclusion that with the symmetry of space and time, the relationship between the coordinates and the time of the same events in two inertial reference systems - fixed  $O_1x_1y_1z_1$  and moving  $O_2x_2y_2z_2$ , shown in Fig. 1, on the basis of the formulas (34)-(38) and (255), should have read:

$$x_1 = x_2 + (V \cdot t) \quad (275)$$

$$x_2 = x_1 - (V \cdot t) \quad (276)$$

$$y_1 = y_2 \quad (36)$$

$$z_1 = z_2 \quad (37)$$

$$t_1 = t_2 = t \quad (277)$$

That is, the Galilean transformation (the system of the equations (35), (36) and (275)-(277)) are true to the significance of any values of the speed  $V$  of the movement of the inertial reference system.

### 3.3. Example № 4, confirming the conclusions, when considering example № 3

Let's consider the following example to also reach the conclusions of example № 3.

In example № 4, as opposed to example № 3, we will consider, not the curved movement of the bodies constituting the closed mechanical system, but the linear movement of bodies, constituting a closed-circuit mechanical system.

Assume that there are two inertial reference system similar to the reference system, shown in Fig. 1, fixed  $O_1x_1y_1z_1$  and moving  $O_2x_2y_2z_2$ , at the speed  $V$  parallel to axis  $O_1x_1$  relative to the system  $O_1x_1y_1z_1$ .

Assume that there is a closed mechanical system of bodies, shown in Fig. 10, and composed of body 1 and body 2, with equal mass  $M_0$  in the state of the rest, and spring 3.

Bodies 1 and 2 are connected to the absolutely elastic spring 3 with no mass (mass which is negligible compared to the masses of bodies 1 and 2).

Under the action of spring 3, bodies 1 and 2 commit a symmetrical reciprocated movement on the common center of the mass of the system - the point  $S$ .

The center of mass of body 1 - the point  $S_1$  and the center of mass of body 2 - the point  $S_2$  reside on a single straight line passing through  $S$ ,  $S_1$  and  $S_2$ .

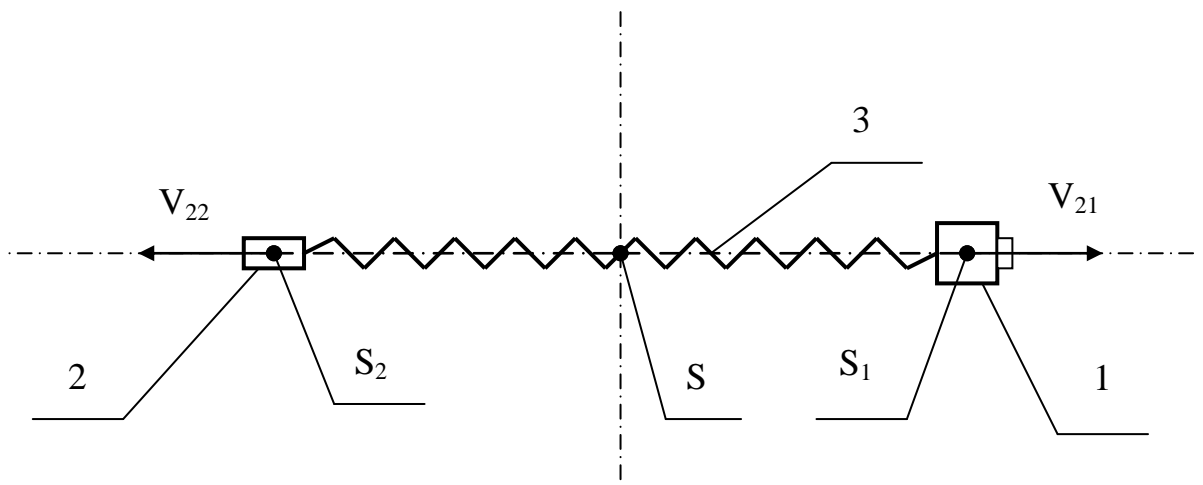


Fig. 10

We will put the closed mechanical system of bodies 1 and 2 with spring 3 in the moving system of reference  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , so that point  $\mathbf{S}$  would be fixed in this reference system and would coincide with the beginning  $\mathbf{O}_2$  of the coordinates. The points  $\mathbf{S}_1$  and  $\mathbf{S}_2$  are on axis  $\mathbf{O}_2\mathbf{x}_2$ , as shown in Fig. 11-13.

In the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , bodies 1 and 2 commit a symmetrical recurrent movement through time  $\mathbf{T}_2$  (the period of fluctuations of the system of bodies 1 and 2).

Suppose, as shown in Fig. 11, that at the time of starting ( $t_2=0$ ) in the reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , spring 3 is fully compressed (spring 3 is at the maximum value of the potential energy of compression), bodies 1 and 2 are in a state of rest, with point  $\mathbf{S}_1$  coinciding with the point  $\mathbf{S}_2$ , the point  $\mathbf{S}$  at the beginning  $\mathbf{O}_2$  of the coordinates (let's say, achieved constructively).

After time  $t_2=0$ , spring 3 begins to uncoil and elbow bodies 1 and 2 in different directions. That is, the potential energy of spring 3 is beginning to change into the kinetic energy of bodies 1 and 2 (the speeds  $\mathbf{V}_{21}$  and  $\mathbf{V}_{22}$  of the movement of the bodies 1 and 2, will gradually rise).

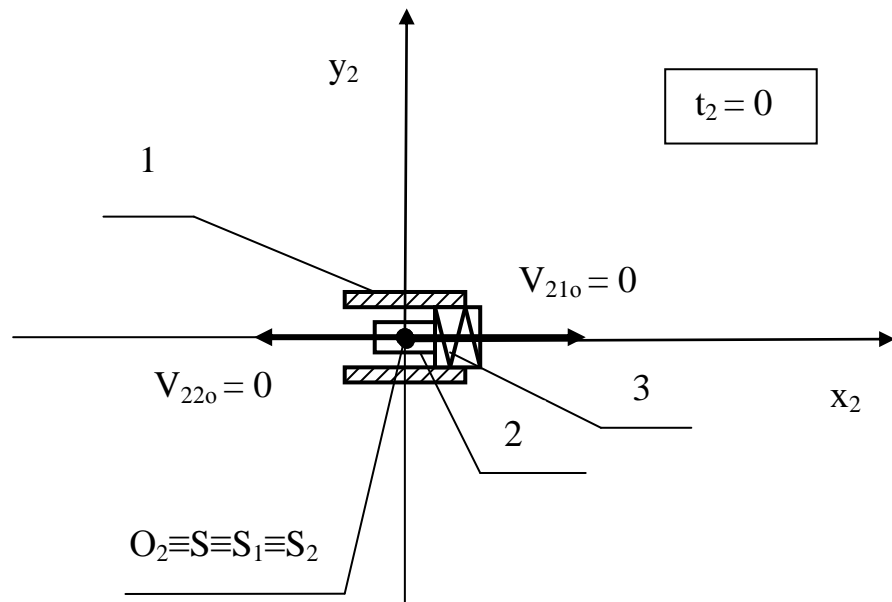


Fig. 11

The reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  at time  $t_2$  equals  $t_{2M0}$  has spring 3 completely uncoiled (the potential energy of spring 3 will be zero). Bodies 1 and 2 will have



the maximum values  $V_{21M}$  and  $V_{22M}$  of speed and the maximum kinetic energy (as shown in Fig. 12).

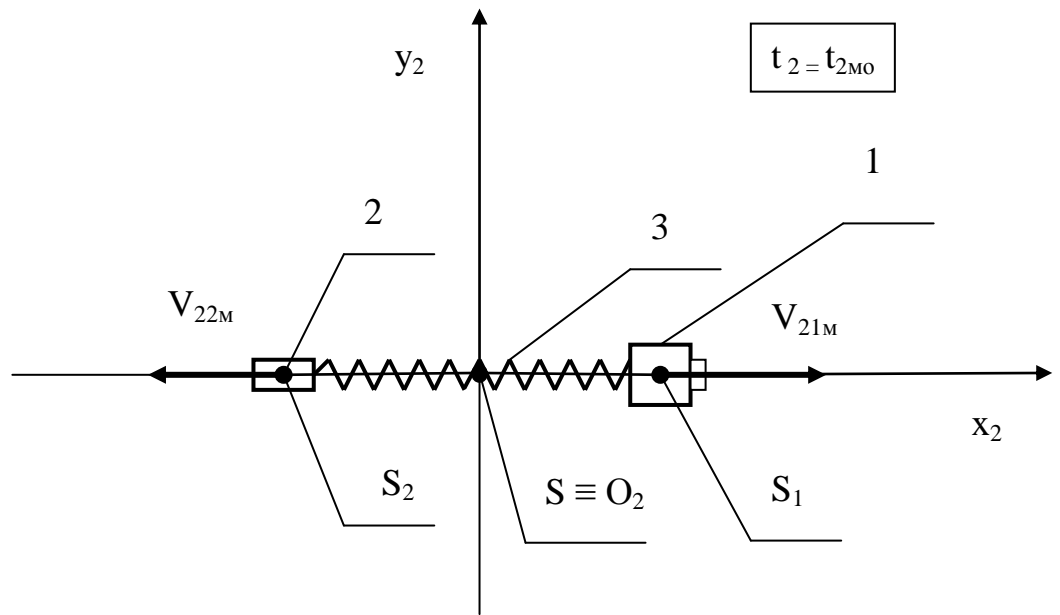


Fig. 12

After time  $t_{2M0}$ , spring 3 begins to be stretched, but bodies 1 and 2 are beginning to slow down, because the kinetic energy is exchanged for the potential energy of the increase in spring 3.

In the reference system  $O_2x_2y_2z_2$  at some point in time  $t_2$  equal to  $t_{2T0}$ , bodies 1 and 2 stop (the kinetic energy of bodies 1 and 2 is zero), but spring 3 is fully extended (the kinetic energy of bodies 1 and 2 has moved fully into the potential energy of the spring, which at the time  $t_{2T0}$  reaches its peak), as shown in Fig. 13.

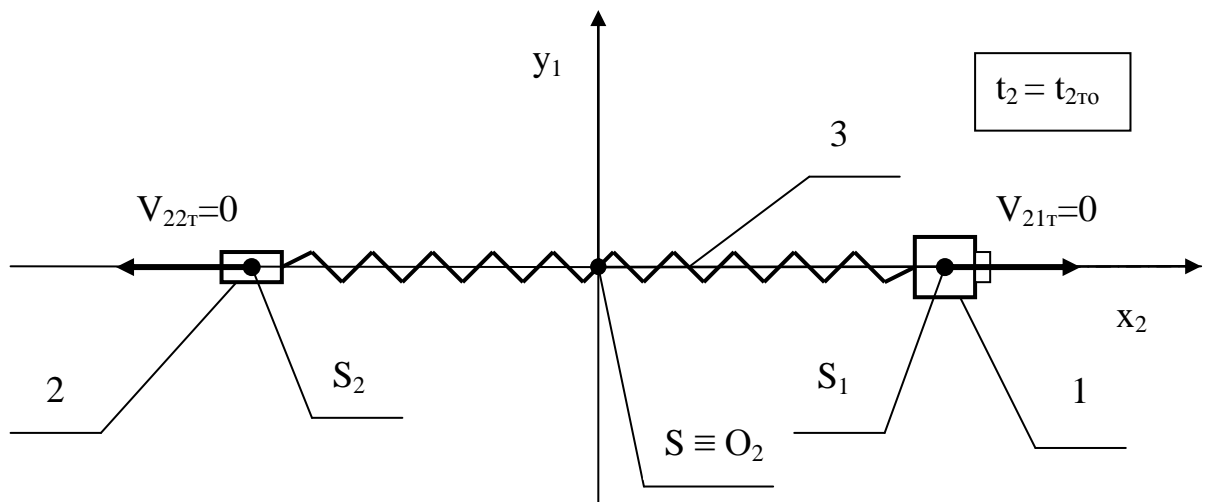


Fig. 13

Further, from time  $t_{2T0}$  before time  $t_2$ , equal to period  $T_2$  of the fluctuation, the process of interaction of bodies 1 and 2 with spring 3 will be opposite (that is, the spring will initially compress itself, conveying its potential energy of the tensile into the kinetic energy of the bodies 1 and 2, and then be compressed under the influence of the bodies 1 and 2, which will convert their kinetic energy into the potential energy of the compression of the spring 3).

Given the frequency of the movement of the bodies 1 and 2 (and spring 3), it may be noted that:

- the position and the condition of bodies 1 and 2 and spring 3, relevant to time  $t_2 = 0$ , will be for times  $t_{2p}$ , equal:

$$t_{2p} = T_2 \cdot n \quad (278)$$

where:  $n = 0, 1, 2, 3, 4 \dots$ ;

- the position and the condition of bodies 1 and 2 and the spring 3, relevant to time  $t_{2M0}$ , will be for times  $t_{2M}$ , equal:

$$t_{2M} = t_{2M0} + (T_2 \cdot n) \quad (279)$$

- the position and the condition of bodies 1 and 2 and spring 3, relevant to time  $t_{2T0}$ , will be for the times  $t_{2T}$ , equal:

$$t_{2T} = t_{2T0} + (T_2 \cdot n) \quad (280)$$

To simplify for further consideration, assume that bodies 1 and 2 are point bodies.

In the moving reference system  $O_2x_2y_2z_2$ , on the basis of symmetry (at any time  $t_2$  the masses of bodies 1 and 2 are equal, the center  $S$  of masses of bodies 1 and 2 coincide with the beginning of the coordinate  $O_2$ ), for any time  $t_2$  the relationship between the coordinate  $x_{21}$  of body 1 and the coordinate  $x_{22}$  of body 2 are inscribed as follows:

$$x_{21} = -x_{22} \quad (281)$$

but the relationship between the speed  $V_{21}$  of body 1 and speeds  $V_{22}$  of body 2 will look:

$$V_{21} = -V_{22} \quad (282)$$

If looking at the movement of the system of the bodies 1 and 2 and spring 3 in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and moving inertial reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , as shown in Fig. 14, then based on equations (34) and (35), you can write the relationship between coordinate  $x_{11}$  of body 1 at time  $t_{11}$  in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and coordinate  $x_{21}$  of body 1 at time  $t_{21}$  in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ :

$$x_{11} = \beta \cdot [x_{21} + (V \cdot t_{21})] \quad (283)$$

$$x_{21} = \beta \cdot [x_{11} - (V \cdot t_{11})] \quad (284)$$

Similarly, using equations (34) and (35), you can write the relationship between coordinate  $x_{12}$  of body 2 at time  $t_{12}$  in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and coordinate  $x_{22}$  of body 2 at time  $t_{22}$  in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ :

$$x_{12} = \beta \cdot [x_{22} + (V \cdot t_{22})] \quad (285)$$

$$x_{22} = \beta \cdot [x_{12} - (V \cdot t_{12})] \quad (286)$$

Using formula (38) you can write the relationship between times  $t_{11}$ ,  $t_{21}$  and  $t_{12}$ ,  $t_{22}$ :

$$t_{11} = \frac{(\beta^2 - 1) \cdot x_{21}}{\beta \cdot V} + (\beta \cdot t_{21}) \quad (287)$$

$$t_{12} = \frac{(\beta^2 - 1) \cdot x_{22}}{\beta \cdot V} + (\beta \cdot t_{22}) \quad (288)$$

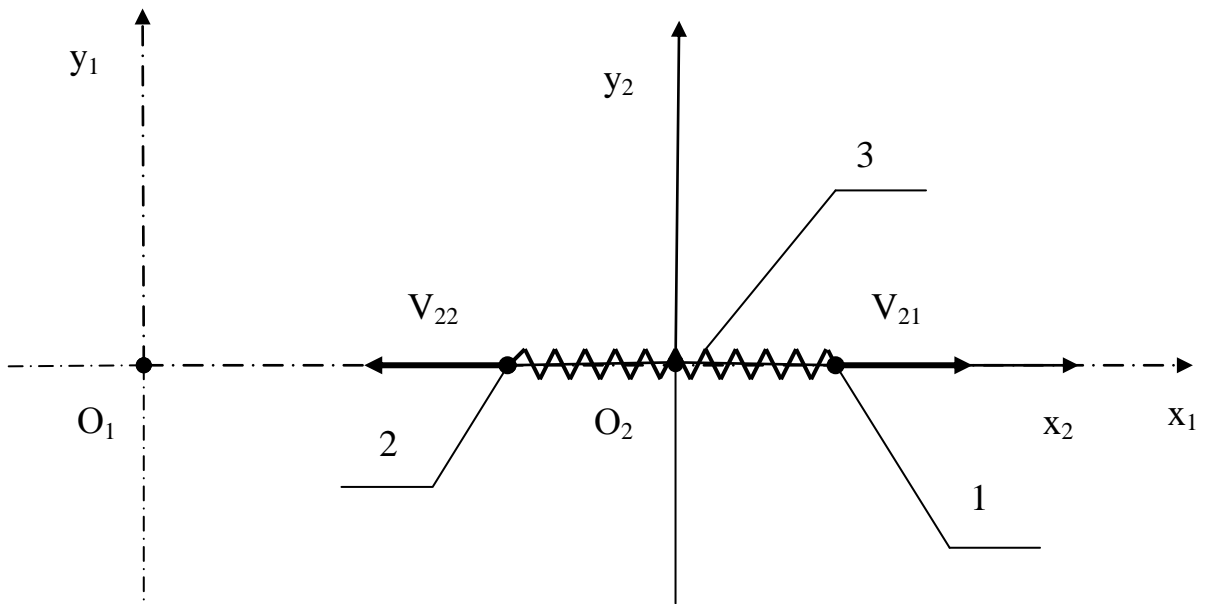


Fig. 14

In the given example, we will be interested in the position of bodies 1 and 2 in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at the same time. That is, when:

$$t_{11} = t_{12} \quad (289)$$

Then equation (289), taking into account formulas (287) and (288) will look like this:

$$\frac{(\beta^2 - 1) \cdot (x_{21} - x_{22})}{\beta^2 \cdot V} = (t_{22} - t_{21}) \quad (290)$$

In the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  in fulfilling conditions (289) the position of bodies 1 and 2 is interesting, when:

$$t_{21} = t_{22} = t_{2p} \quad (291)$$

Substituting condition (291) in equation (290), we get:

$$x_{21} = x_{22} = 0 \quad (292)$$

That is, to fulfill the conditions (289) and (291) bodies 1 and 2 (the mass centers) in the reported time should be at the point which coincides with the center of mass  $\mathbf{S}$  of bodies 1 and 2 and the beginning  $\mathbf{O}_2$  of the coordinates.

Hence:

$$t_{2p} = T_2 \cdot n \quad (278)$$

where:  $n = 0, 1, 2, 3, 4 \dots$

Given, that  $x_{21} \geq 0$  and  $x_{22} \leq 0$  (the baseline condition), for the case when  $t_{21} \neq t_{2p}$  and  $t_{22} \neq t_{2p}$ , from formula (290) shows, that time  $t_{22}$  depending on the value of the transition coefficient  $\beta$  can be:

$$- \quad t_{22} > t_{21} \quad \text{with } \beta > 1 ; \quad (293)$$

$$- \quad t_{22} < t_{21} \quad \text{with } 0 < \beta < 1 ; \quad (294)$$

$$- \quad t_{22} = t_{21} \quad \text{with } \beta = 1 . \quad (295)$$

Now we may proceed to verify the implementation of the law of conservation of momentum.

Consider two points in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$ .

### 3.3.1.1. Time $t_{1p}$

As shown in Fig. 15, in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  at time  $t_2$ , equal  $t_{2p}$ , bodies 1 and 2 are at one point, which coincides with center  $\mathbf{O}_2$  of the

coordinates  $\mathbf{O}_2$  (the baseline condition), and their speed  $V_{21p}$  and  $V_{22p}$  respectively, equal:

$$V_{21p} = 0 \quad (296)$$

$$V_{22p} = 0 \quad (297)$$

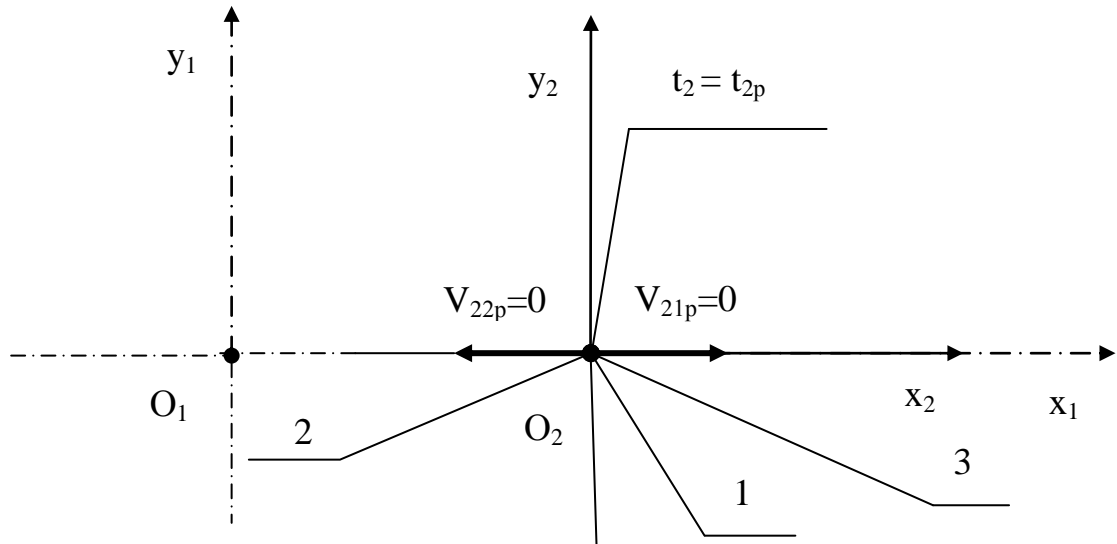


Fig. 15

Assuming that in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  at the time  $t_{2p}$  bodies 1 and 2 are at one point (that is, the coordinates  $\mathbf{x}_{21p}$  and  $\mathbf{x}_{22p}$  of bodies 1 and 2 are equal), in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  bodies 1 and 2 at time  $t_{1p}$ , related to time  $t_{2p}$  in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ , also will be at one point (that is, the coordinates  $\mathbf{x}_{11p}$  and  $\mathbf{x}_{12p}$  of bodies 1 and 2 are equal).

Thus, in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at time  $t_{1p}$ , bodies 1 and 2 are at one point and speeds  $V_{11p}$  and  $V_{12p}$  respectively, taking into account formula (40) and equality (301) and (302) are equal:

$$V_{11p} = V \quad (298)$$

$$V_{12p} = V \quad (299)$$

Consequently, in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  the momentum  $\mathbf{P}_{1p}$  of the closed mechanical system of bodies 1 and 2 (and spring 3) at time  $t_{1p}$  taking into account formula (152) and the equality (303) and (304) equals:

$$P_{11p} + P_{12p} = P_{1p} = \frac{2 \cdot M_0 \cdot V}{\sqrt{1 - \frac{V^2}{V_{\text{кр}}^2}}} \quad (300)$$

### 3.3.1.2. Time $t_{1T}$

As discussed earlier, in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  at time  $t_2$  equal to  $t_{2T}$ , which for body 1 save, as  $t_{21T}$ , body 1 is the speed  $V_{21T}$  equal to zero:

$$V_{21T} = 0 \quad (301)$$

because spring 3 at the time  $t_2$ , equals  $t_{21T}$ , is the maximum energy potential of increase.

The provision of body 1 in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  at time  $t_{21T}$  will be consistent with the position of body 1 in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at time  $t_{1T}$ .

In the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at time  $t_{1T}$  body 1, according to equation (40), will be equal to speed  $V_{11T}$ :

$$V_{11T} = V \quad (302)$$

In the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at time  $t_1$  equal  $t_{1T}$  body 2 will have a speed equal to  $V_{12T}$ .

The position of body 2 in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  at time  $t_{1T}$  will be consistent with the position of body 2 in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  at time  $t_2$ , which is equal to  $t_{22T}$ .

As shown in Fig. 16, assume that in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  at time  $t_2$  equal  $t_{22T}$  body 2 has a speed equal to  $V_{22T}$ .

Given condition (293), that in the transition coefficient  $\beta > 1$  time  $t_{22} > t_{21}$ , it may be noted, that speed  $V_{22T}$  of body 2 will be directed towards axis  $\mathbf{O}_2\mathbf{x}_2$ .

In addition, on the basis of conditions (294), claims that, when the transition coefficient  $0 < \beta < 1$  time  $t_{22} < t_{21}$ , it may be noted that, when the transition coefficient  $0 < \beta < 1$  speed  $V_{22T}$  of body 2 will be in the direction opposite the direction of axis  $\mathbf{O}_2\mathbf{x}_2$ .

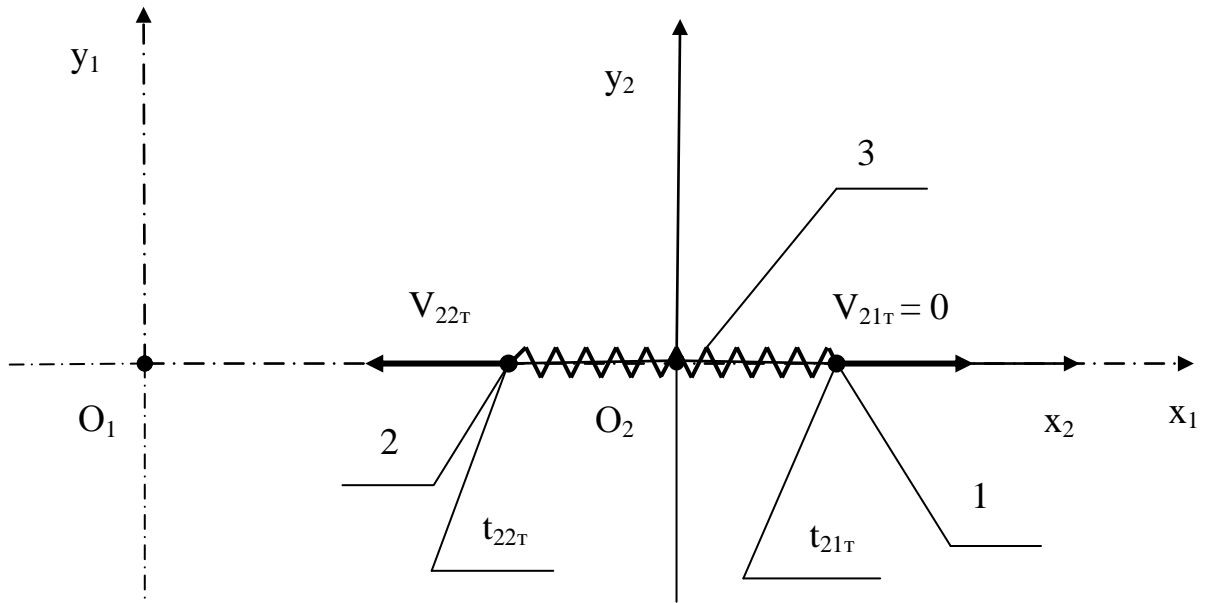


Fig. 16

Using formula (40), you can write the relationship between speeds  $V_{12T}$  and  $V_{22T}$  of body 2:

$$V_{12T} = \frac{V_{22T} + V}{\frac{(\beta^2 - 1) \cdot V_{22T}}{\beta^2 \cdot V} + 1} \quad (303)$$

Consequently, in the fixed reference system  $O_1x_1y_1z_1$  momentum  $P_{1T}$  of the closed mechanical system of bodies 1 and 2 (and spring 3) at time  $t_{1T}$  taking into account formula (147) and equity (302) equals:

$$P_{11T} + P_{12T} = P_{1T} = \frac{M_0 \cdot V}{\sqrt{1 - \frac{V^2}{V_{xkp}^2}}} + \frac{M_0 \cdot V_{12T}}{\sqrt{1 - \frac{V_{12T}^2}{V_{xkp}^2}}} \quad (304)$$

### 3.3.1.3. Determination of the environment in which the law of conservation of momentum is maintained for example № 4

Due to the fact, that the mechanical system of bodies 1 and 2 (and spring 3) is closed, the law of conservation of momentum allows writing for times  $t_{1p}$  and  $t_{1T}$  in the fixed reference system  $O_1x_1y_1z_1$ , the following equation:

$$P_{1T} = P_{1p}$$

Or, based on formulas (300) and (304):

$$\frac{M_0 \cdot V}{\sqrt{1 - \frac{V^2}{V_{\text{кр}}^2}}} + \frac{M_0 \cdot V_{12\tau}}{\sqrt{1 - \frac{V_{12\tau}^2}{V_{\text{кр}}^2}}} = \frac{2 \cdot M_0 \cdot V}{\sqrt{1 - \frac{V^2}{V_{\text{кр}}^2}}} \quad (305)$$

From equation (305) show, that the necessary condition (the value of the speed  $V_{12\tau}$ ), which in example № 4 will be implemented the law of conservation of momentum in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  is:

$$V_{12\tau} = V \quad (306)$$

or, given formula (303):

$$V_{22\tau} = 0 \quad (307)$$

From equations (306) and (307), show that speeds  $V_{12\tau}$  and  $V_{22\tau}$  are not dependent on speed  $V$  (and hence, not dependent on the value of the transition coefficient  $\beta$ ).

But in the moving reference system  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$  when coordinate  $\mathbf{x}_{21\tau}$  of body 1 and  $\mathbf{x}_{22\tau}$  of body 2 are not equal to zero, speed  $V_{22\tau} = 0$  of body 2 is possible only when:

$$t_{22\tau} = t_{21\tau} \quad (308)$$

By placing equity (308) in formula (290), we get:

$$\frac{(\beta^2 - 1) \cdot (\mathbf{x}_{21} - \mathbf{x}_{22})}{\beta^2 \cdot V} = 0 \quad (309)$$

But since value  $(\mathbf{x}_{21} - \mathbf{x}_{22}) > 0$ , then from equation (309) there will be another condition for implementation of the law of conservation of momentum in the fixed reference system  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}$  for example № 4:

$$\beta = 1 \quad (259)$$

Thus, it can be concluded that in a closed mechanical system of bodies considered in example № 4, for values of the transition coefficient, located in the ranges  $\beta > 1$  and  $0 < \beta < 1$ , the law of conservation of momentum is not fulfilled.



#### IV. The inference

The conclusions of the above can be summarized.

##### The kinematics

Using the principle of relativity and allowing the symmetry of space and time:

1. Turning from the system of equations for the relationship between inertial reference systems - fixed  $\mathbf{O}_1\mathbf{x}_1\mathbf{y}_1\mathbf{z}_1$  and moving  $\mathbf{O}_2\mathbf{x}_2\mathbf{y}_2\mathbf{z}_2$ :

$$x_1 = \beta_1 \cdot [x_2 + (V_1 \cdot t_2)] \quad (24)$$

$$x_2 = \beta_2 \cdot [x_1 + (V_2 \cdot t_1)] \quad (25)$$

$$y_1 = \beta_3 \cdot y_2 \quad (26)$$

$$y_2 = \beta_4 \cdot y_1 \quad (27)$$

$$z_1 = \beta_5 \cdot z_2 \quad (28)$$

$$z_2 = \beta_6 \cdot z_1 \quad (29)$$

to the system of the equations:

$$x_1 = \beta \cdot [x_2 + (V \cdot t_2)] \quad (34)$$

$$x_2 = \beta \cdot [x_1 - (V \cdot t_1)] \quad (35)$$

$$y_1 = y_2 \quad (36)$$

$$z_1 = z_2 \quad (37)$$

2. To establish, that the values of the transition coefficient  $\beta$  for the inertial reference systems can be in two mutually exclusive ranges:

- $\beta > 1$ ,
- $0 < \beta < 1$

3. To get the formula for the transition coefficient  $\beta$  for the inertial reference systems in the case of  $\beta > 1$ :

$$\beta_{>}^2 = \frac{1}{1 - \frac{V^2}{v_{\text{кр}1}^2}} \quad (59)$$

where:  $v_{\text{кр}1}$  - the constant actual value;

4. To get the formula for the transition coefficient  $\beta$  for inertial reference systems for the case of  $0 < \beta < 1$ :

$$\beta_{<}^2 = \frac{1}{1 + \frac{V^2}{v_{\text{xkp2}}^2}} \quad (60)$$

where:  $v_{\text{xkp2}}$  - the constant actual value;

5. To establish, that when the transition coefficient  $\beta > 1$ , there has been only the real value  $V_{\text{xkp}}$  (equal  $v_{\text{xkp1}}$ ) of the speed of the point, which to be invariant in the all directions and in the all inertial reference systems:

$$v_{\text{xkp1}} = \text{Const} \quad (71)$$

6. To establish, that when the transition coefficient  $0 < \beta < 1$ , there has been only the perceived value  $V_{\text{xkp}}$  (equal  $(\mathbf{i} \cdot \mathbf{v}_{\text{xkp2}})$ ) of the speed of the point, which to be invariant in the all directions and in the all inertial reference systems:

$$v_{\text{xkp2}} = \text{Const} \quad (72)$$

### The dynamics

1. Using compliance with the law of conservation of momentum and the law of conservation of mechanical energy (more precisely, the special case, when the constancy of the potential energy - the constancy of the kinetic energy) in the inertial reference systems for the closed mechanical system of bodies, moving in line and experiencing only absolutely elastic interactions, were obtained the accord of masses, of momentum and of kinetic energy of the body at its speed:

- when  $\beta > 1$ :

$$M(V)_{>} = \frac{M_0}{\sqrt{1 - \frac{V^2}{v_{\text{xkp1}}^2}}} \quad (150)$$

$$P(V)_{>} = \frac{M_0 \cdot V}{\sqrt{1 - \frac{V^2}{v_{\text{xkp1}}^2}}} \quad (151)$$

$$E_k(V)_> = M_0 \cdot v_{xkp1}^2 \cdot \left( \frac{1}{\sqrt{1 - \frac{V^2}{v_{xkp1}^2}}} - 1 \right) \quad (152)$$

- when  $0 < \beta < 1$ :

$$M(V)_< = \frac{M_0}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \quad (171)$$

$$P(V)_< = \frac{M_0 \cdot V}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \quad (172)$$

$$E_k(V)_< = M_0 \cdot v_{xkp2}^2 \cdot \left( 1 - \frac{1}{\sqrt{1 + \frac{V^2}{v_{xkp2}^2}}} \right) \quad (173)$$

2. In a separate example (example № 3), in which was considered the closed mechanical system of bodies, not moving linearly, it was shown that when the values of the transition coefficient  $\beta$  are in the ranges  $\beta > 1$  and  $0 < \beta < 1$ , there is the violation of the law of the conservation of the momentum, the kinetic energy, and angular momentum (when there is constancy in the potential energy of the system). That is, the momentum, angular momentum and kinetic energy of a closed mechanical system have values that are variables in time.

The relationship between the laws of conservation of momentum, kinetic energy and angular momentum of a closed mechanical system and the symmetry of space and time (the homogeneity and isotropy of space and homogeneity of time) enables us to note, that when the values of the transition coefficient  $\beta$  in ranges  $\beta > 1$  and  $0 < \beta < 1$ , the condition of symmetry of space and of time is violated.

When considering example № 3, it has been shown that the laws of conservation of momentum, kinetic energy and angular momentum of a closed mechanical system, and hence the condition of symmetry of space and time are met only if the transition coefficient is  $\beta = 1$ .

But bearing in mind that the condition of symmetry of space and time is the requirement (the baseline condition) of the special theory of relativity for space and time, the conclusions of the special theory of relativity are in contradiction with the condition of the symmetry of space and time, established at its creation, suggests the following:

**- the relationship between the coordinates and time in inertial reference systems can be recorded using the special theory of relativity, if the values of the transition coefficient  $\beta$  are in the ranges  $\beta > 1$  or  $0 < \beta < 1$ ;**

**- in the one directed inertial reference system, the transition coefficient  $\beta$  can not be more or less than 1. It can only be equal to 1;**

**- in the inertial reference systems, the transition coefficient  $\beta$  does not depend on speed  $V$ ;**

**- the Galileo transformation is true for inertial reference systems for any values of speed  $V$ :**

$$\mathbf{x}_1 = \mathbf{x}_2 + (\mathbf{V} \cdot t) \quad (275)$$

$$\mathbf{x}_2 = \mathbf{x}_1 - (\mathbf{V} \cdot t) \quad (276)$$

$$y_1 = y_2 \quad (36)$$

$$z_1 = z_2 \quad (37)$$

$$t_1 = t_2 = t \quad (277)$$

It also should be noted, that the conclusions reached in the chapter "the dynamics" are only true when the potential energy of the body does not depend on its speed (that is, its kinetic energy).

P.S.: The main ideas contained in the article "The special theory of the relativity without the postulate of the constancy of the speed of the light", printed in the magazine "The actual problems of the modern science" (ISSN 1680-2721) № 1

(34) for the 2007 year and located on the sites " The new ideas and the hypothesis" <http://new-idea.kulichki.net/?mode=physics> and "The mathematical physics. The theory of relativity" <http://www.matphysics.ru/>.

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