Using the Law of Conservation of Momentum to Test the Validity of SRT

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This article attempts to show that the use of the special relativity theory (SRT), when considering the motion of a closed mechanical system in the inertial reference systems, can lead to non-compliance with the law of conservation of momentum. PACS number: 03.30.+p

1. Introduction

Special Relativity Theory (SRT) can be divided into relativistic kinematics and relativistic dynamics. Relativistic *kinematics* establishes a connection (Lorentz transformations) between the coordinates and time of an event, occurring at a point of space, in one inertial reference system, and coordinates and time of the same event in another inertial reference system, and the relationship between the values of projections of speeds of the point (conversion of the speeds) at appropriate times in two inertial reference systems. Relativistic *dynamics*, based on the mandatory implementation of the laws of conservation of momentum and energy for a closed system of bodies whose interaction is instantaneous, establishes the dependences of mass and momentum of the point material body on its speed in inertial reference systems.

This article suggests an analysis with the following steps:

1) Take a closed mechanical system of bodies whose interaction will be permanent;

2) Select two inertial reference systems, mobile and immobile, with respect to the center of mass of the closed system of bodies;

3) Select two points in time in the mobile inertial reference system;

4) With the help of the Lorentz transformation, determine the positions of bodies at the selected points in time in the mobile reference system;

5) Using the conversion speeds, determine the projections of speeds of bodies in these moments of time in the mobile reference system;

6) Knowing the values of projections of speeds of the bodies and using the dependences of mass and momentum of a body on the speed, determine the values of the momentums of the bodies at selected points in time in the mobile reference system;

7) Write the law of conservation of momentum in the mobile reference system at the two selected points in time, and determine the conditions for its implementation.

2. The Main Dependences of SRT

Assume that there are two inertial reference systems, shown in Fig.1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$, in which:

- Similar the axis of the Cartesian coordinate systems $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$ are pairs parallel and equally directed;

- System $O_2x_2y_2z_2$ moves relative to the system $O_1x_1y_1z_1$ with constant speed V along the axis O_1x_1;

- In both systems, the start times (t_1 = 0 and t_2 = 0) are selected when the origin O_1 and O_2 of these systems are identical.

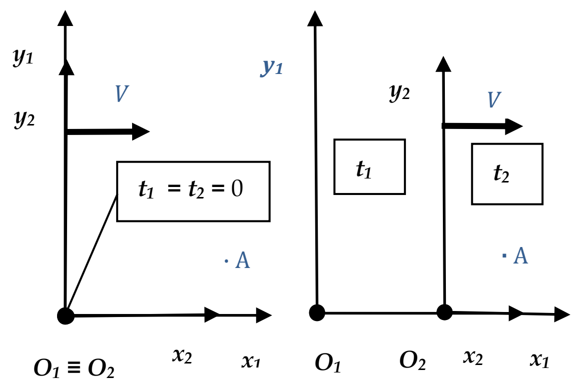

Figure 1.

In SRT, Lorentz transformation [1] gives the relationship between the spatial coordinates x_1, y_1, z_1 of point A at time t_1 in a stationary inertial reference system $O_1x_1y_1z_1$, and coordinates x_2, y_2, z_2 of the same point A in the mobile inertial reference system $O_2x_2y_2z_2$, at the time t_2 corresponding to time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$. The spatial part of the Lorentz transformation goes as follows:

$$x_1 = [x_2 + (V \cdot t_2)] \,/\, \sqrt{1 - V^2 / c^2} \qquad (1)$$

$$x_2 = [x_1 - (V \cdot t_1)] \,/\, \sqrt{1 - V^2 / c^2} \qquad (2)$$

$$y_1 = y_2 \qquad (3)$$

$$z_1 = z_2 \qquad (4)$$

where: c is the speed of light in a vacuum.

From formulas (1) and (2) we can write the dependence for times t_1 and t_2:

$$t_1 = [t_2 + (V \cdot x_2 / c^2)] \,/\, \sqrt{1 - V^2 / c^2} \qquad (5)$$

$$t_2 = [t_1 - (V \cdot x_1 / c^2)] \,/\, \sqrt{1 - V^2 / c^2} \qquad (6)$$

Also in SRT, conversion of the speeds [1] - the relationship between the projections v_{x1}, v_{y1} and v_{z1} of the velocity of a point on the axis of the Cartesian coordinates at time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$ and similar projections v_{x2}, v_{y2} and v_{z2} of the velocity of the same point in the mobile inertial reference system $O_2x_2y_2z_2$ at time t_2, corresponding to time t_1 in the stationary inertial reference system $O_1x_1y_1z_1$, is written as:

$$v_{x1} = (v_{x2} + V) \,/\, \{1 + [(V \cdot v_{x2})/c^2]\} \qquad (7)$$

$$v_{x2} = (v_{x1} - V) / \{1 - [(V \cdot v_{x1})/c^2]\} \quad (8)$$

$$v_{y1} = \left(v_{y2} \cdot \sqrt{1 - V^2 / c^2}\right) / \{1 + [(V \cdot v_{x2}) / c^2]\} \quad (9)$$

$$v_{y2} = \left(v_{y1} \cdot \sqrt{1 - V^2 / c^2}\right) / \{1 - [(V \cdot v_{x1}) / c^2]\} \quad (10)$$

…

The dependence of the mass $M(v)$ and the momentum $P(v)$ of a moving body, having a rest mass M_0, on the speed v in SRT take the forms [1]:

$$M(v) = M_0 \cdot \sqrt{1 - V^2 / c^2} \quad (11)$$

$$\bar{P}(v) = M_0 \cdot \bar{v} \cdot \sqrt{1 - V^2 / c^2} \quad (12)$$

3. Description of a Closed Mechanical System of Bodies

For consideration we take the simplest closed mechanical system of the bodies, which have constant interaction. Assume that there are two inertial reference systems, similar to those of reference systems, shown in Fig.1, stationary $O_1x_1y_1z_1$ and mobile $O_2x_2y_2z_2$, which moves relative to the system $O_1x_1y_1z_1$ with speed V parallel to the axis O_1x_1.

Suppose that there is a closed mechanical system of bodies as shown in Fig. 2, consisting of point bodies, Body 1 and Body 2, with equal mass M_0 at rest, and a String 3.

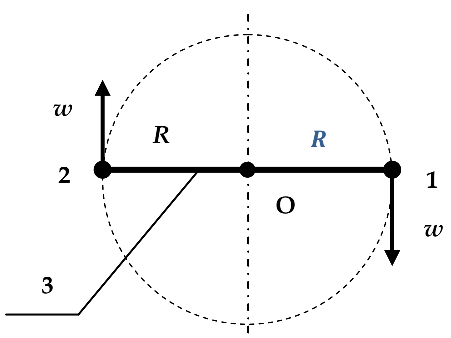

Figure 2.

In Fig. 2, Bodies 1 and 2 are connected with String 3, the mass of which can be neglected because of its smallness. Bodies 1 and 2 rotate with angular speed w around a common center of mass - the point O. The distance from the point Body 1 (Body 2) to point O is equal to R.

Let us put the closed mechanical system of Bodies 1 and 2, with the String 3, in the moving reference system $O_2x_2y_2z_2$, so that the point O would be stationary in this reference system, and coincident with the origin O_2, and let the rotation of Bodies 1 and 2 around it occur in a clockwise direction in the plane of $O_2x_2y_2z_2$, as shown in Fig. 3.

Also assume, that at the start of timing ($t_2 = 0$) in the reference system $O_2x_2y_2z_2$, Bodies 1 and 2 were on the axis O_2x_2, with Body 1 had a positive coordinate, and Body 2 had a negative coordinate.

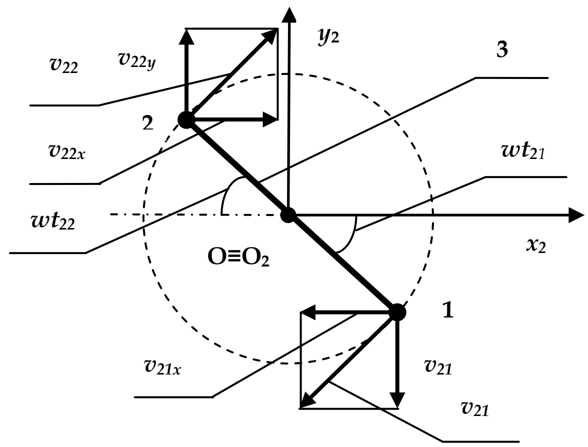

Figure 3.

In the mobile reference system $O_2x_2y_2z_2$ at any time t_2, Bodies 1 and 2 will have the speeds v_{21} and v_{22}, equal to v_R:

$$v_{21} = v_{22} = v_R = \omega \cdot R \quad (13)$$

In this case, the projections v_{21x} and v_{21y} of speed of Body 1 and the projections v_{22x} and v_{22y} of speed of Body 2 on the axis O_2x_2 and O_2y_2, respectively, for times t_{21} and t_{22} will be:

$$v_{21x} = - v_R \cdot \sin(\omega \cdot t_{21}) \quad (14)$$

$$v_{21y} = - v_R \cdot \cos(\omega \cdot t_{21}) \quad (15)$$

$$v_{22x} = v_R \cdot \sin(\omega \cdot t_{22}) \quad (16)$$

$$v_{22y} = v_R \cdot \cos(\omega \cdot t_{22}) \quad (17)$$

The relationship between the coordinates x_{21} and y_{21} of the Body 1 depending on time t_{21} and the relationship between the coordinates x_{22} and y_{22} of Body 2 depending on the time t_{22} in the mobile reference system $O_2x_2y_2z_2$ can be written as:

$$x_{21} = R \cdot \cos(\omega \cdot t_{21}) \quad (18)$$

$$y_{21} = - R \cdot \sin(\omega \cdot t_{21}) \quad (19)$$

$$x_{22} = - R \cdot \cos(\omega \cdot t_{22}) \quad (20)$$

$$y_{22} = R \cdot \sin(\omega \cdot t_{22}) \quad (21)$$

Based on the Eqs. (1) and (3), we can write the relationships between:

- coordinates x_{11} and y_{11} of Body 1 at time t_{11} in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{21} and y_{21} of the Body 1 in the mobile reference system $O_2x_2y_2z_2$ at time t_{21}, which corresponds to the time t_{11} in the stationary reference system $O_1x_1y_1z_1$:

$$x_{11} = [x_{21} + (V \cdot t_{21})] / \sqrt{1 - V^2 / c^2} \quad (22)$$

$$y_{11} = y_{21} \quad (23)$$

- coordinates x_{12} and y_{12} of Body 2 at time t_{12} in the stationary reference system $O_1x_1y_1z_1$ and coordinates x_{22} and y_{22} of Body 2 in the mobile reference system $O_2x_2y_2z_2$ at time t_{22}, which corresponds to the time t_{12} in the stationary reference system $O_1x_1y_1z_1$:

$$x_{12} = [x_{22} + (V \cdot t_{22})] / \sqrt{1 - V^2 / c^2} \quad (24)$$

$$y_{12} = y_{22} \quad (25)$$

Using formula (5), the relationship between the values of the times t_{11} and t_{21}, t_{12} and t_{22} will look like this:

$$t_{11} = \{t_{21} + [(V \cdot x_{21}) / c^2]\} / \sqrt{1 - V^2 / c^2} \quad (26)$$

$$t_{12} = \{t_{22} + [(V \cdot x_{22}) / c^2]\} / \sqrt{1 - V^2/c^2} \quad (27)$$

Suppose we are interested in the position of Bodies 1 and 2 in the stationary reference system $O_1x_1y_1z_1$ at the same time, *i.e.* where:

$$t_{11} = t_{12} \quad (28)$$

Taking into account formulas (18), (20), (26) and (27), Eq. (28) becomes:

$$t_{21} + [V \cdot R \cdot \cos(\omega \cdot t_{21}) / c^2] = \\ = t_{22} - [V \cdot R \cdot \cos(\omega \cdot t_{22}) / c^2] \quad (29)$$

Now for consideration, select two points in time in the stationary reference system $O_1x_1y_1z_1$.

4. A Moment of time t_{1P}

In the mobile reference system $O_2x_2y_2z_2$, under condition (28), what are the positions of Bodies 1 and 2 at a time t_{2P}, when:

$$t_{21} = t_{22} = t_{2p} \quad (30)$$

Substituting condition (30) in Eq. (29) for the case, when $\omega \cdot t_{2p} < \pi$, we obtain:

$$\omega \cdot t_{2p} = \pi / 2 \quad (31)$$

That is, as shown in Fig. 4, under the terms of (28), (30) and (31) in the moving mobile reference system $O_2x_2y_2z_2$ at time t_{2P}, Body 1 and Body 2 are on a line parallel to the axis O_2y_2 and in the stationary reference system $O_1x_1y_1z_1$, Body 1 and Body 2 will be on a line parallel to the axis O_1y_1 at time t_{11} (t_{12}), equal t_{1P} and which corresponds to the time t_{2P} in the mobile reference system $O_2x_2y_2z_2$.

According to Eqs. (31), (14-17), in the mobile reference system $O_2x_2y_2z_2$ at time t_{2P}, Body 1 and Body 2, respectively, have the following values of the projections v_{21xP} , v_{21yP} and v_{22xP} , v_{22yP} of speeds of his movement on the axis O_2x_2 and O_2y_2:

$$v_{21xp} = - v_R \quad (32)$$

$$v_{21yp} = 0 \quad (33)$$

$$v_{22xp} = v_R \quad (34)$$

$$v_{22yp} = 0 \quad (35)$$

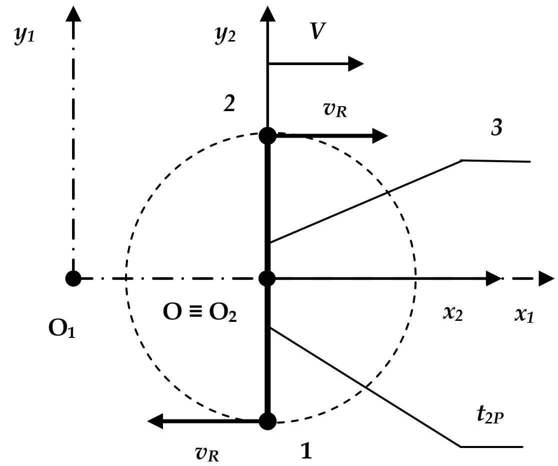

Figure 4.

Then, on the basis of formulas (7), (9) and equalities (32-35), in the stationary reference system $O_1x_1y_1z_1$ at time t_{1P} Body 1 and Body 2, respectively, will have the following values of the projections v_{11xP} , v_{11yP} and v_{12xP} , v_{12yP} of speeds of his movement on the axis O_1x_1 and O_1y_1:

$$v_{11xp} = (V - v_R) / \{1 - [(V \cdot v_R) / c^2]\} \quad (36)$$

$$v_{11yp} = 0 \quad (37)$$

$$v_{12xp} = (V + v_R) / \{1 + [(V \cdot v_R)/c^2]\} \quad (38)$$

$$v_{12yp} = 0 \quad (39)$$

Hence, using formulas (11) and (12), it may be noted that in the stationary reference system $O_1x_1y_1z_1$ at time t_{1P} the Body 1 and the Body 2, respectively, will have the following values of the projections P_{11xP}, P_{11yP} and P_{12xP}, P_{12yP} of momentums on the axis O_1x_1 and O_1y_1:

$$P_{11xp} = (M_o \cdot v_{11xp}) / \sqrt{1 - (v_{11x}^2 / c^2)} \quad (40)$$

$$P_{12xp} = (M_o \cdot v_{12xp}) / \sqrt{1 - (v_{12x}^2 / c^2)} \quad (41)$$

$$P_{11yp} = 0 \quad (42)$$

$$P_{12yp} = 0 \quad (43)$$

5. Moment of time t_{1h}

Also in the mobile reference system $O_2x_2y_2z_2$ when performing the condition (28) it is interesting position of Body 2 when finding the Body 1 on the axis O_2x_2 at time t_{21}, equal to t_{2h}, where:

$$t_{21h} = 0 \quad (44)$$

The value of time t_{22}, when performing the conditions (28) and (44), denote t_{22h}, for which the Eq. (29) becomes:

$$\omega \cdot t_{22h} = \{v_R \cdot V \cdot [1 + \cos(\omega \cdot t_{22h})]\} / c^2 \quad (45)$$

As can be seen from equation (45), the value of time t_{22h} must be greater than zero.

Under the terms of (28) and (44) in the mobile reference system $O_2x_2y_2z_2$ at time $t_{21h} = 0$ the Body 1 will be located on the axis O_2x_2, and in the stationary reference system $O_1x_1y_1z_1$ the Body 1 will be located on the axis O_1x_1 at time t_{11} (t_{12}), equal t_{1h} and which corresponds to the time $t_{21h} = 0$ in the mobile reference system $O_2x_2y_2z_2$.

Moreover in the mobile reference system $O_2x_2y_2z_2$ according to Eq. (45), the Body 2 cannot be on the axis O_2x_2 at time t_{22}, equal t_{22h} and which corresponds to the time t_{1h} in the stationary reference system $O_1x_1y_1z_1$.

That is, as shown in Fig. 5, the Body 1 is located on the axis O_1x_1 in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h}, which corresponds to the time $t_{21h} = 0$ in the mobile reference system $O_2x_2y_2z_2$, and at the time t_{1h} the Body 2 cannot lie on the axis O_2x_2.

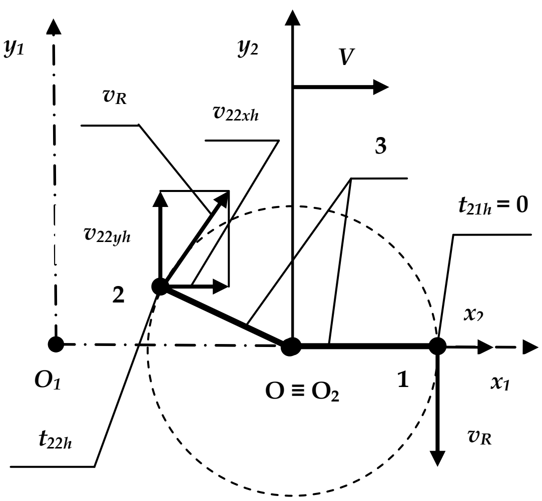

Figure 5.

In the mobile reference system $O_2x_2y_2z_2$ the Body 1 at the time $t_{21h} = 0$ and the Body 2 at the time t_{22h} respectively have projections v_{21xh}, v_{21yh} and v_{22xh}, v_{22yh} of speeds of his movement on the axis O_2x_2 and O_2y_2, so that:

$$v_{21xh} = 0 \qquad (46)$$

$$v_{21yh} = -\, v_R \qquad (47)$$

Then, on the basis of formulas (7), (9) and Eqs. (46), (47), in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} the Body 1 and the Body 2, respectively, will have the values of the projections v_{11xh}, v_{11yh} and v_{12xh}, v_{12yh} of speeds of his movement on the axis O_1x_1 and O_1y_1:

$$v_{11xh} = V \qquad (48)$$

$$v_{11yh} = -\, v_R \cdot \sqrt{1 - V^2 / c^2} \qquad (49)$$

$$v_{12xh} = (V + v_{22xh}) \,/\, \{1 + [(V \cdot v_{22xh}) \,/\, c^2]\} \qquad (50)$$

$$v_{12yh} = \left[v_{22yh} \cdot \sqrt{1 - (V^2/c^2)}\right] / \{1 + [(V \cdot v_{22xh})/c^2]\} \; (51)$$

Given Eq. (45), we note that the time $t_{22h} > 0$, so the projection v_{22yh} of the speed will be the direction of the axis O_2x_2.

From Eqs. (16) and (17) it follows that:

$$v_{22xh}{}^2 + v_{22yh}{}^2 = v_R{}^2 \qquad (52)$$

Using formulas (11) and (12), may be noted that in the stationary reference system $O_1x_1y_1z_1$ at time t_{1h} the Body 1 and the Body 2, respectively, will have the following values of the projections P_{11xh}, P_{11yhP} and P_{12xh}, P_{12yh} of momentums on the axis O_1x_1 and O_1y_1:

$$P_{11xh} = (M_o \cdot v_{11xh}) \,/\, \sqrt{1 - \left[\left(v_{11xh}^2 + v_{11yh}^2\right) / \, c^2\right]} \; (53)$$

$$P_{12xh} = (M_o \cdot v_{12xh}) \,/\, \sqrt{1 - \left[\left(v_{12xh}^2 + v_{12yh}^2\right) / \, c^2\right]} \; (54)$$

$$P_{11yh} = \left(M_o \cdot v_{11yh}\right) / \sqrt{1 - \left[\left(v_{11xh}^2 + v_{11yh}^2\right) / \, c^2\right]} \; (55)$$

$$P_{12yh} = \left(M_o \cdot v_{12yh}\right) / \sqrt{1 - \left[\left(v_{12xh}^2 + v_{12yh}^2\right) / \, c^2\right]} \; (56)$$

6. Verification of the Law of Conservation of Momentum

The law of conservation of momentum of a closed mechanical system of bodies, connected with the symmetry properties of space - the homogeneity of space [1], states, that the momentum of a closed mechanical system of bodies (which is not acted upon by external forces) is a constant value, *i.e.* in any inertial reference system for any point in time the value of the momentum of a closed mechanical system of bodies is a constant value (because there is no external influence).

Due to the fact, that the mechanical system of the Bodies 1 and 2 (and String 3) is closed, the law of conservation of momentum can write the following equations for the moments of times t_{1P} and t_{1h}:

$$P_{11xp} + P_{12xp} = P_{11xh} + P_{12xh}$$

$$P_{11yp} + P_{12yp} = P_{11yh} + P_{12yh}$$

or:

$$\left[(M_o \cdot v_{11xp})/\sqrt{1 - (v_{11x}^2/c^2)}\right] +$$

$$+ \left[(M_o \cdot v_{12xp})/\sqrt{1 - (v_{12x}^2/c^2)}\right] =$$

$$= \left\{(M_o \cdot v_{11xh})/\sqrt{1 - \left[\left(v_{11xh}^2 + v_{11yh}^2\right)/c^2\right]}\right\} +$$

$$+ \left\{(M_o \cdot v_{12xh})/\sqrt{1 - \left[\left(v_{12xh}^2 + v_{12yh}^2\right)/c^2\right]}\right\} \qquad (57)$$

$$0 + 0 = \left\{(M_o \cdot v_{11yh})/\sqrt{1 - \left[\left(v_{11xh}^2 + v_{11yh}^2\right)/c^2\right]}\right\} +$$

$$+ \left\{(M_o \cdot v_{12yh}) \,/\, \sqrt{1 - \left[\left(v_{12xh}^2 + v_{12yh}^2\right)/c^2\right]}\right\} \qquad (58)$$

By inserting the projections v_{11xP}, v_{12xP}, v_{11xh}, v_{11yh}, v_{12xh} and v_{12yh} of speeds of formulas (36), (38), (48) - (51) in equations (57) and (58) and using the formula (52), we obtain:

$$\left\{[M_o \cdot (V - v_R)]/\left[\sqrt{[1-(v_R{}^2/c^2)]\cdot[1-(V^2/c^2)]}\,\right]\right\} +$$

$$+\left\{[M_o \cdot (V + v_R)]/\left[\sqrt{[1-(v_R{}^2/c^2)]\cdot[1-(V^2/c^2)]}\right]\right\} =$$

$$=\left\{(M_o \cdot V)/\left[\sqrt{[1-(v_R{}^2/c^2)]\cdot[1-(V^2/c^2)]}\right]\right\} +$$

$$+\left\{[M_o \cdot (V + v_{22xh})] /\right.$$

$$\left. / \sqrt{[1-(v_R{}^2/c^2)]\cdot[1-(V^2/c^2)]}\right\} \qquad (59)$$

$$0 = -\left\{(M_o \cdot v_R)/\sqrt{[1-(v_R{}^2/c^2)]}\right\} +$$

$$+\left\{\left(M_o \cdot v_{22yh}\right) / \sqrt{[1-(v_R{}^2/c^2)]}\right\} \qquad (60)$$

or:

$$V - v_R + V + v_R = V + V + v_{22xh} \qquad (61)$$

$$0 = -v_R + v_{22yh} \qquad (62)$$

From Eqs. (61) and (62) obtain the necessary conditions (the values of the projections v_{22xh} and v_{22yh} of speeds), which in this example will be implemented by law of conservation of momentum in the stationary inertial reference system $O_1x_1y_1z_1$:

$$v_{22xh} = 0 \qquad (63)$$

$$v_{22yh} = v_R \qquad (64)$$

Substituting conditions (63) and (64) in Eqs. (16) and (17), we obtain:

$$t_{22h} = t_{21h} = 0 \qquad (65)$$

And substituting Eq. (65) in the formula (45):

$$\omega \cdot 0 = [(v_R \cdot V) / c^2] \cdot [1+1] \qquad (66)$$

will have another condition for the implementation of the law of conservation of momentum in the stationary inertial reference system $O_1x_1y_1z_1$ for considered example:

$$0 = 1 / c^2 \qquad (67)$$

But since the speed of light c is not infinite, so the condition (67) is not feasible, and therefore in this case, the law of conservation of momentum cannot be implemented.

That is, we can conclude, that in the stationary inertial reference system $O_1x_1y_1z_1$ the application of the special theory of relativity to describe the motion of a closed mechanical system of bodies, considered in this example, leads to non-compliance of the law of conservation of momentum.

9. Conclusion

It can be concluded, that the use of the special theory of relativity in dealing with individual examples may lead to non-compliance with the law of conservation of momentum for a closed mechanical system in the inertial reference systems.

Given, that the law of conservation of momentum associated with the homogeneity of space [1], we can assume, that the failure of the law of conservation of momentum will lead to non-compliance with conditions of symmetry of space and time, on which is based the special theory of relativity.

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