The special theory of relativity: linear example of infringement of laws of preservation of an impulse

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In article attempt to show becomes that use of the special theory of relativity can lead to infringement of the law of preservation of an impulse of the closed mechanical system consisting of bodies, located on one line and which interaction has constant character, in inertial systems of readout.

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The maintenance

1. Introduction

As shown in [1] on an example of the flat closed mechanical system of the bodies which interaction has constant character, application of the special theory of a relativity can lead to that in inertial system of readout the impulse and kinetic energy of the closed mechanical system will be variables on time in sizes.

It is offered to consider possibility of infringement of the law of preservation of an impulse at the linear closed mechanical system of the bodies which interaction has constant character, in inertial system of readout.

2. The description of the closed mechanical system of bodies

For consideration we take the elementary closed mechanical system of the bodies testing constant interaction and making rectilinear movements.

Let's assume that there is the closed mechanical system of bodies, shown on fig. 1 and consisting of spring 3 and bodies 1 and 2, having equal weight M_0 at rest.

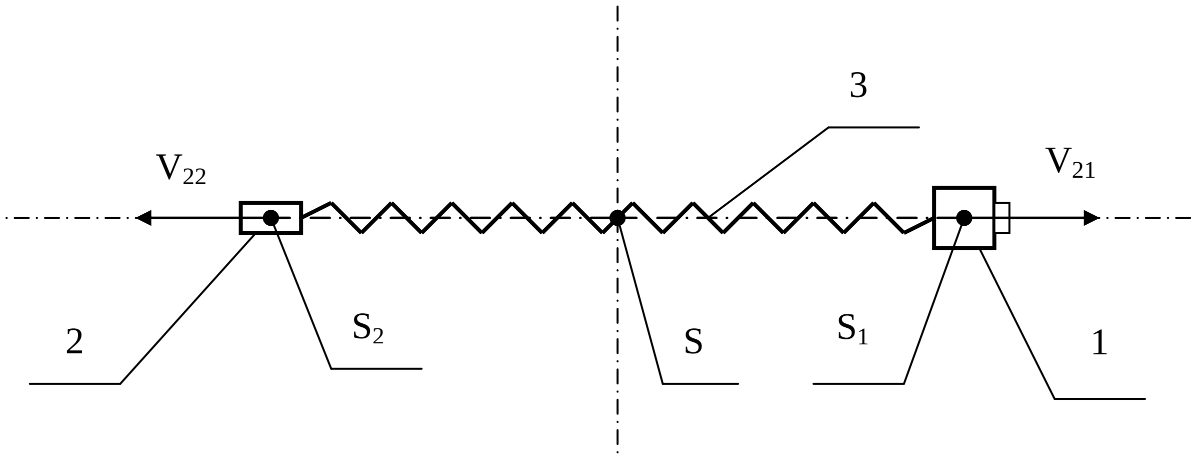

Fig. 1

Bodies 1 and 2 are connected to absolutely elastic spring 3, which weight infinitesimal in comparison with weights of bodies 1 and 2.

Under the influence of a spring 3 bodies 1 and 2 make symmetric linear back and forth motions concerning the general center of weights of system of bodies 1 and 2 - point S.

The center of weights of a body 1 - point S_1 and the center of weights of a body 2 - point S_2 constantly are on one straight line passing through points S, S_1 and S_2.

Let's place the considered closed mechanical system of bodies 1 and 2 with a spring 3 in inertial system of readout $O_2x_2y_2z_2$ so that point S would be motionless in this system of readout and coincided with the beginning of coordinates O_2, and points S_1 and S_2 would be on axis O_2x_2, as is shown in fig. 2.

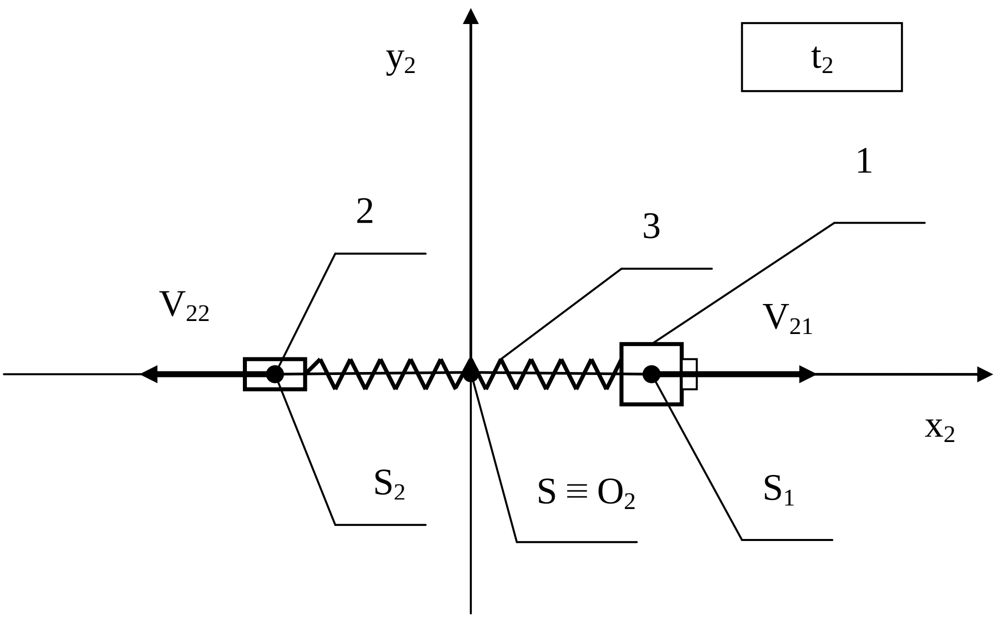

Fig. 2

In inertial system of readout $O_2x_2y_2z_2$ a body 1 and 2 make the symmetric movements periodically repeating through time t_{2n} (the period of fluctuation of system of bodies 1 and 2).

Let's assume that at the moment of time reference mark (t_2=0) in system of readout $O_2x_2y_2z_2$ the spring 3 is completely compressed, bodies 1 and 2 are at rest, and point S_1 coincides with point S_2, point S and the beginning of coordinates O_2 (we will admit that have achieved it structurally).

Dependences of coordinates x_{21} and x_{22} of positions of the centers of weights S_1 and S_2 and speeds of movement V_{21} and V_{22} of bodies 1 and 2 accordingly from time t_2 in inertial system of readout $O_2x_2y_2z_2$ are represented on fig. 3.

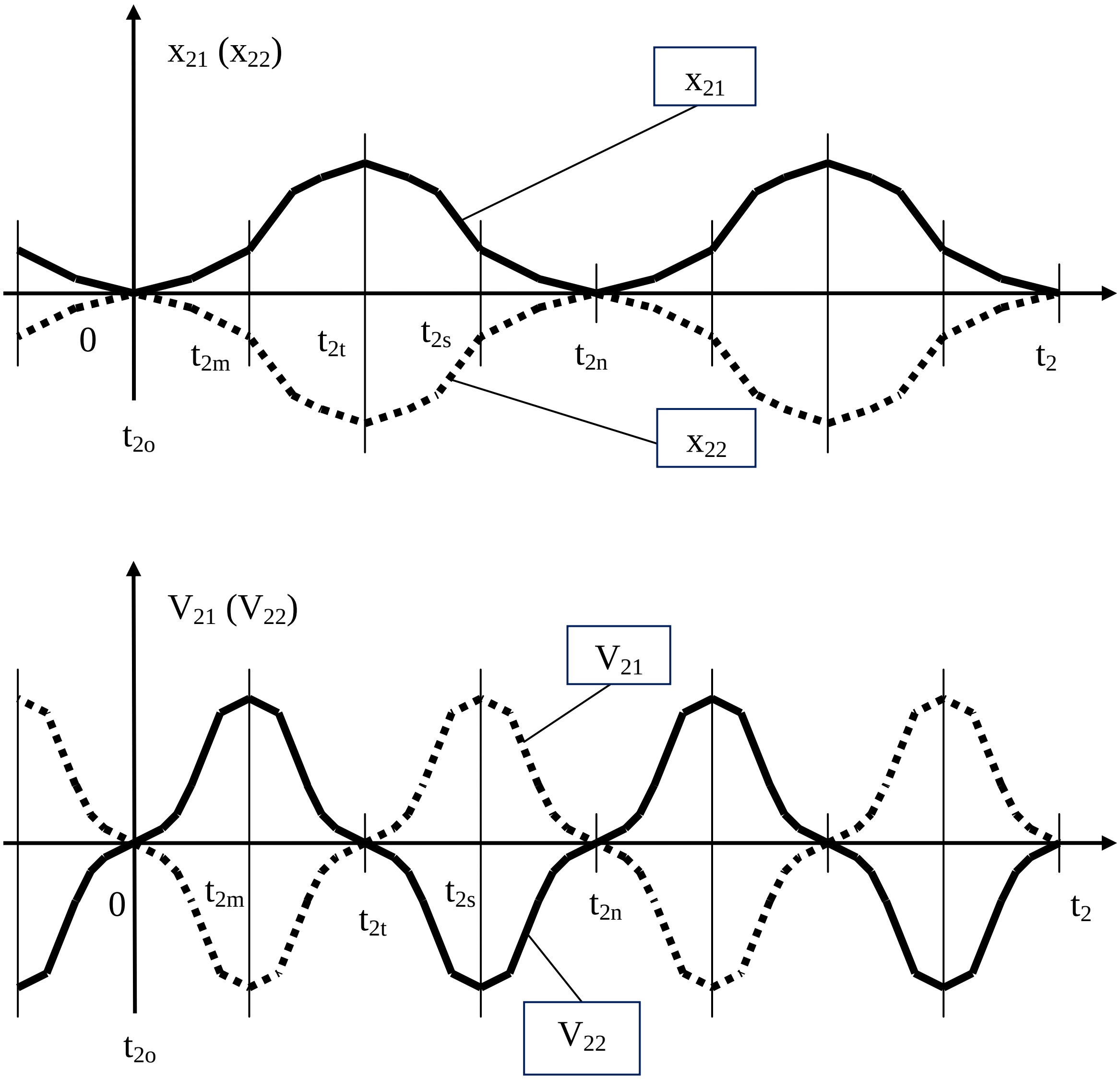

Fig. 3

At the moment of time reference mark t_{2o}, when $t_2=0$, in inertial system of readout $O_2x_2y_2z_2$: the spring 3 is completely compressed (a spring 3 has the maximum value of potential energy of compression), bodies 1 and 2 are at rest, point S_1 coincides with points S_2 and S and the beginning of coordinates O_2.

From the moment of time t_{2o} on time moment t_{2m}: the spring 3 is unclenched and parts forcibly bodies 1 and 2 every which way, potential energy of compression of a spring 3 passes in kinetic energy of bodies 1 and 2 (absolute sizes of speeds V_{21} and V_{22} of movements of bodies 1 and 2 accordingly will gradually increase).

At the moment of time t_{2m}: the spring 3 will be completely unclenched (potential energy of a spring 3 will be equal to zero), bodies 1 and 2 will have maximum on absolute size of speed of movements and the maximum values of kinetic energy.

From the moment of time t_{2m} on time moment t_{2t}: the spring 3 is stretched, and bodies 1 and 2 are slowed down, kinetic energy of bodies 1 and 2 pass in potential energy of a stretching of a spring 3.

At the moment of time t_{2t}: bodies 1 and 2 stop (kinetic energy of bodies 1 and 2 are equal to zero), and the spring 3 is completely stretched (kinetic energy of bodies 1 and 2 have passed completely in potential energy of a stretching of a spring 3 which at the moment of time t_{2t} reaches the maximum value).

From the moment of time t_{2t} on time moment t_{2s}: the spring 3 is compressed, and bodies 1 and 2 are accelerated, potential energy of a stretching of a spring 3 passes in kinetic energy of bodies 1 and 2.

At the moment of time t_{2s}: the spring 3 will be completely unclenched (potential energy of a spring 3 will be equal to zero), bodies 1 and 2 will have the maximum on absolute of size speeds V_{21} and V_{22} of movements and the maximum values of kinetic energy.

From the moment of time t_{2s} on time moment t_{2n}: the spring 3 is compressed, and bodies 1 and 2 are slowed down, kinetic energy of bodies 1 and 2 pass in potential energy of compression of a spring 3.

At the moment of time t_{2n}: the spring 3 is completely compressed (a spring 3 has the maximum value of potential energy of compression), bodies 1 and 2 are at rest, point S_1 coincides with points S_2 and S and the beginning of coordinates O_2.

For simplification of the further consideration we will assume that bodies 1 and 2 are dot.

In inertial system of readout $O_2x_2y_2z_2$, proceeding from symmetry (at any moment t_2 weights of bodies 1 and 2 are identical, center S of weights of bodies 1 and 2 coincides with the beginning of coordinates O_2), for any moment of time

t_2 communication between coordinate x_{21} of body 1 and coordinate x_{22} of body 2 will register as follows:

$$x_{21} = - x_{22} \quad (1)$$

and communication between speed V_{21} movement of a body 1 and speed V_{22} movement of a body 2 will look like:

$$V_{21} = - V_{22} \quad (2)$$

Let's enter the inertial system of readout $O_1x_1y_1z_1$ shown on fig. 4.

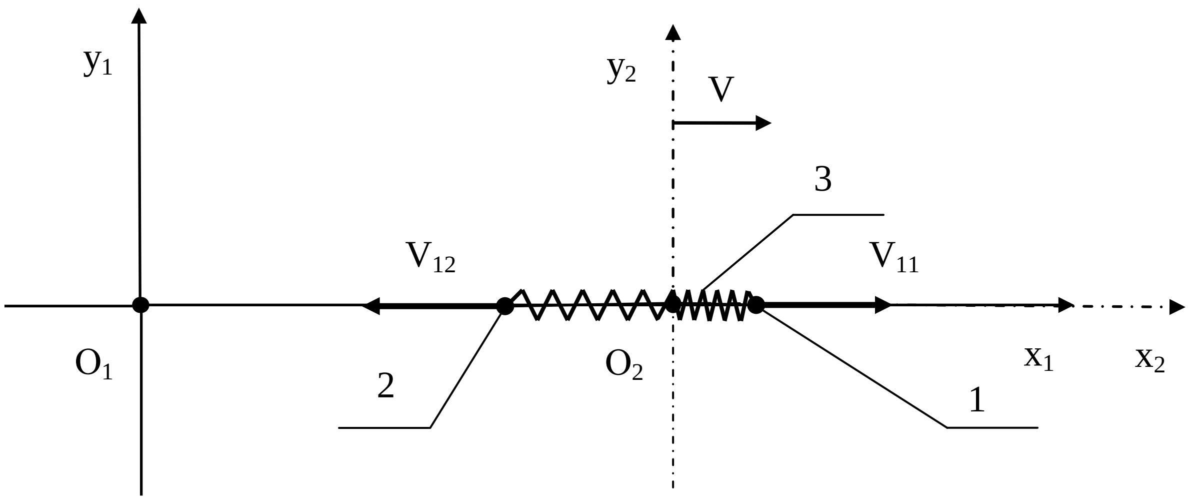

Fig. 4

Let's admit that at inertial systems of readout $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$:

- Similar axes of the Cartesian coordinates are in pairs parallel and equally directed;

- System $O_2x_2y_2z_2$ moves concerning system $O_1x_1y_1z_1$ with constant speed V along axis O_1x_1;

- As time reference mark (t_1=0 and t_2=0) in both systems that moment when the beginnings of coordinates O_1 and O_2 these systems coincided is chosen.

If to consider movement of system of bodies 1 and 2 (and springs 3) in inertial systems of readout $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$ leaning Lorentz's transformations [2], it is possible to write communication between coordinate x_{11} of body 1 at the moment of time t_{11} in system of readout $O_1x_1y_1z_1$ and coordinate x_{21} of body 1 in system of readout $O_2x_2y_2z_2$ at the moment of time t_{21}, corresponding to time moment t_{11} in system of readout $O_1x_1y_1z_1$:

$$x_{11} = \frac{x_{21} + (V \cdot t_{21})}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (3)$$

$$x_{21} = \frac{x_{11} - (V \cdot t_{11})}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (4)$$

where: c – a velocity of light in vacuum.

It is similarly possible to write down communication between coordinate x_{12} of body 2 at the moment of time t_{12} in system of readout $O_1x_1y_1z_1$ and coordinate x_{22} of body 2 in system of readout $O_2x_2y_2z_2$ at the moment of time t_{22}, corresponding to time moment t_{12} in system of readout $O_1x_1y_1z_1$:

$$x_{12} = \frac{x_{21} + (V \cdot t_{22})}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (5)$$

$$x_{22} = \frac{x_{12} - (V \cdot t_{12})}{\sqrt{1 - \frac{V^2}{c^2}}} \quad (6)$$

According to transformations of speeds [2] communication between speed V_{21} of movement of a body 1 at the moment of time t_{21} in system of readout $O_2x_2y_2z_2$ and speed V_{11} of movement of a body 1 in system of readout $O_1x_1y_1z_1$ at the moment of time t_{11}, corresponding to time moment t_{21} in system of readout $O_2x_2y_2z_2$, will look in a following kind:

$$V_{21} = \frac{V_{11} - V}{1 - \frac{V \cdot V_{11}}{c^2}} \quad (7)$$

$$V_{11} = \frac{V_{21} + V}{1 + \frac{V \cdot V_{21}}{c^2}} \quad (8)$$

Similarly communication between speed V_{22} of movement of a body 2 at the moment of time t_{22} in system of readout $O_2x_2y_2z_2$ and speed V_{12} of movement of a body 2 in system of readout $O_1x_1y_1z_1$ at the moment of time t_{12}, corresponding to time moment t_{22} in system of readout $O_2x_2y_2z_2$, will register as:

$$V_{22} = \frac{V_{12} - V}{1 - \frac{V \cdot V_{11}}{c^2}} \qquad (9)$$

$$V_{12} = \frac{V_{22} + V}{1 + \frac{V \cdot V_{22}}{c^2}} \qquad (10)$$

By means of formulas (3) - (6) it is possible to write communication between values of times t_{11}, t_{21} and t_{12}, t_{22}:

$$t_{11} = \frac{t_{21} + \frac{V \cdot x_{21}}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (11)$$

$$t_{21} = \frac{t_{11} - \frac{V \cdot x_{11}}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (12)$$

$$t_{12} = \frac{t_{22} + \frac{V \cdot x_{22}}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (13)$$

$$t_{22} = \frac{t_{12} - \frac{V \cdot x_{12}}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (14)$$

From formulas (12) and (14) it is possible to receive that in case $t_{11} = t_{12}$:

$$t_{21} + \frac{V \cdot x_{21}}{c^2} = t_{22} + \frac{V \cdot x_{22}}{c^2} \qquad (15)$$

Considering that always $x_{21} \geq 0$ and $x_{22} \leq 0$ (the initial condition), from the formula (15) is visible that time size t_{22} should be always more sizes of time t_{21}:

$$t_{21} < t_{22} \qquad (16)$$

for a case, when $t_{11}=t_{12}$, $x_{21} \neq 0$ and $x_{22} \neq 0$.

Dependences of coordinates x_{11} and x_{12} and speeds of movement V_{11} and V_{12} of bodies 1 and 2 accordingly from time t_1 in inertial system of readout $O_1x_1y_1z_1$ are represented on fig. 5.

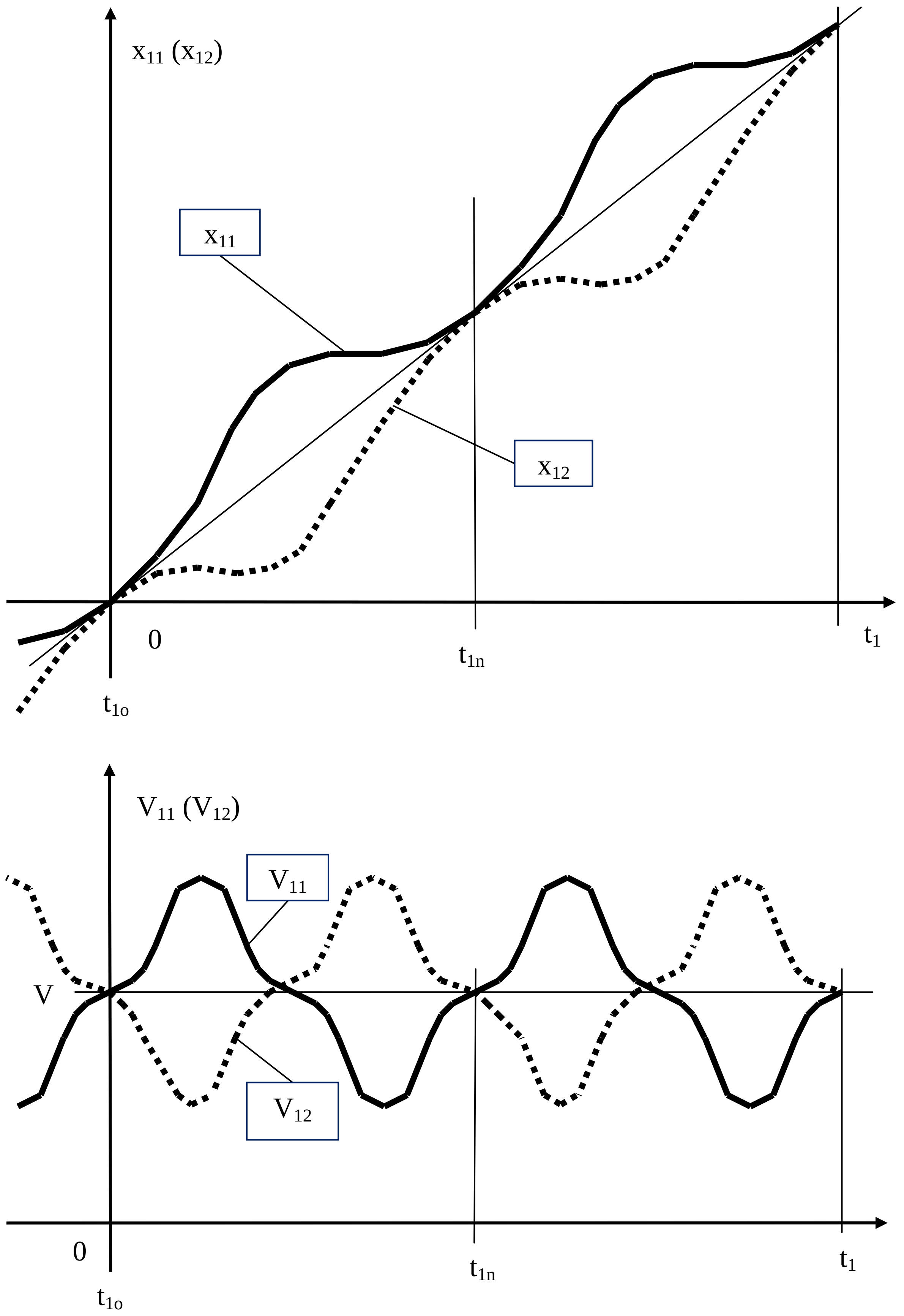

Fig. 5

Knowing dependence of an impulse of a body on speed of its movement [2] it is possible to write down formulas for impulses P_{11} and P_{12} bodies 1 and 2 in inertial system of readout $O_1x_1y_1z_1$:

$$P_{11} = \frac{M_0 \cdot V_{11}}{\sqrt{1 - \frac{{V_{11}}^2}{c^2}}} \qquad (17)$$

$$P_{12} = \frac{M_0 \cdot V_{12}}{\sqrt{1 - \frac{{V_{12}}^2}{c^2}}} \qquad (18)$$

Impulse P_1 of system of bodies 1 and 2 (and springs 3) will be equal in inertial system of readout $O_1x_1y_1z_1$:

$$P_1 = P_{11} + P_{12} = M_0 \cdot \left(\frac{V_{11}}{\sqrt{1 - \frac{{V_{11}}^2}{c^2}}} + \frac{V_{12}}{\sqrt{1 - \frac{{V_{12}}^2}{c^2}}} \right) \qquad (19)$$

Considering fig. 5, dependence of impulse P_1 of system of bodies 1 and 2 (and springs 3) from time t_1 in inertial system of readout $O_1x_1y_1z_1$ will look, as is shown in fig. 6.

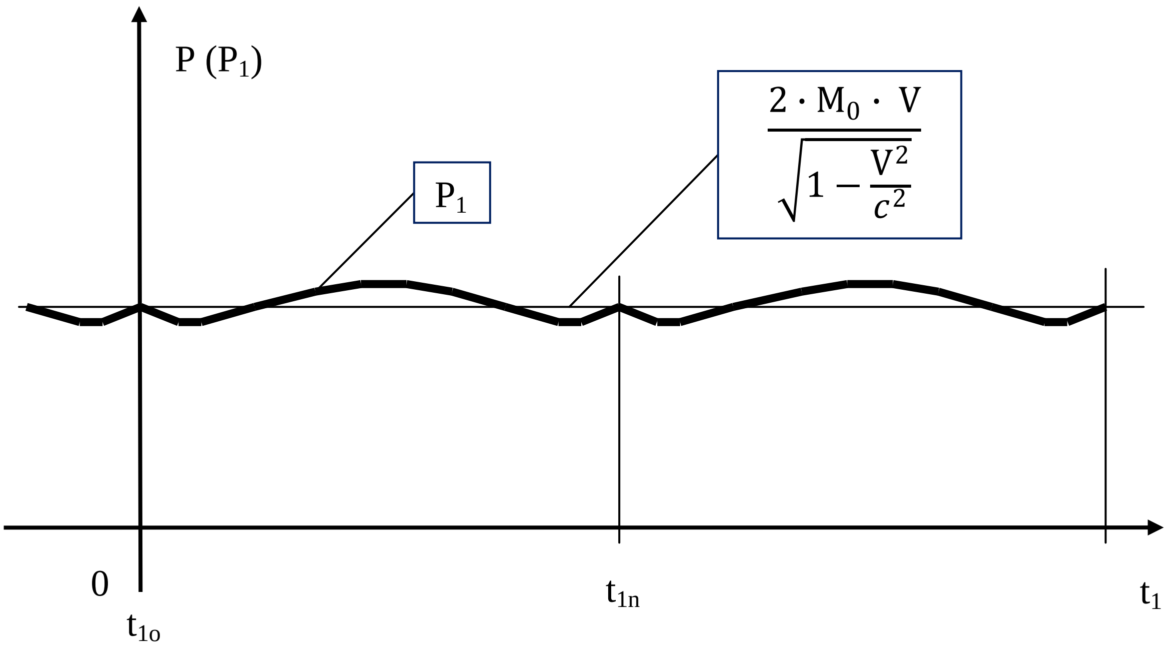

Fig. 6

From fig. 6 it is visible that in inertial system of readout $O_1x_1y_1z_1$ the closed mechanical system of bodies 1 and 2 (and springs 3) has variable in time t_1 on absolute size and a direction of a vector of impulse P_1 (i.e. impulse P_1 of this closed system is time function t_1).

And since in a considered example system of bodies 1 and 2 (and springs 3) – closed, system of readout $O_1x_1y_1z_1$ – inertial, this variability of size of an impulse contradicts the law of preservation of the impulse [2], asserting that in inertial system of readout size of an impulse of the closed mechanical system should be necessarily constant (not to depend on size of the moment of time).

As a result it is possible to draw a conclusion that in inertial system of readout application of the special theory of relativity at the description of movement of the closed mechanical system of the bodies considered in the given example, leads to default of the law of preservation of an impulse.

3. Definition of a condition of performance of the law of preservation of an impulse

To check up the results received above, we will try to define a condition at which in inertial system of readout $O_1x_1y_1z_1$ for the closed system consisting of bodies 1 and 2 (and springs 3) the law of preservation of an impulse will be carried out.

For consideration we will choose in inertial system of readout $O_1x_1y_1z_1$ two moments of time t_1 and we will define a condition providing a constancy of size of an impulse of system of bodies 1 and 2 (and springs 3) for these two moments of time.

As the first moment of time t_1 we will choose the moment of time t_{1o} (when t_{1o} =0).

As is shown in fig. 7, in inertial system of readout $O_1x_1y_1z_1$ at the moment of time t_{1o} bodies 1 and 2 will be in one point (points S_1, S_2, S and the beginning of coordinates O_1 coincide), coordinate x_{11o} of body 1 and coordinate x_{12o} of body 2 will be equal to zero:

$$x_{11o} \ = \ x_{12o} = 0 \qquad (20)$$

speed V_{11o} of body 1 and speed V_{12o} of body 2 will be equal V (speed of movement of system of readout $O_2x_2y_2z_2$ concerning system of readout $O_1x_1y_1z_1$):

$$V_{11o} = V_{12o} = V \qquad (21)$$

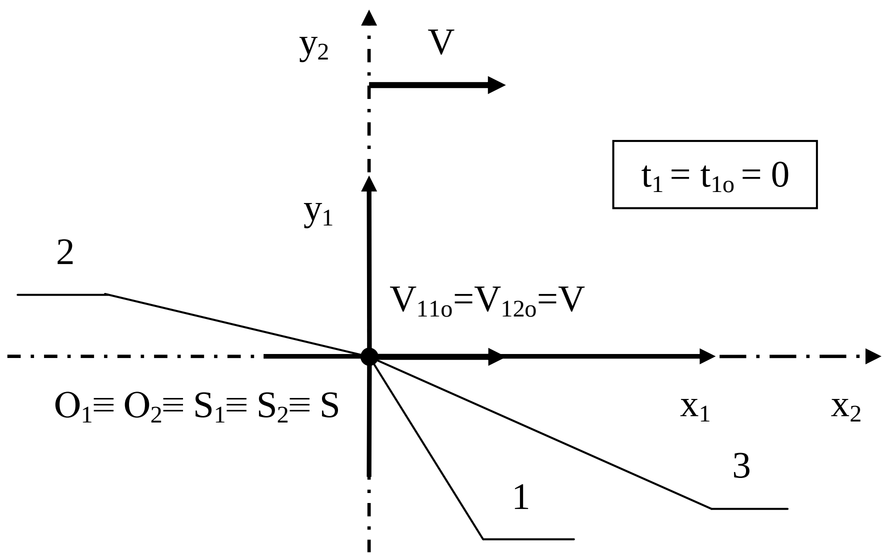

Fig. 7

In inertial system of readout $O_2x_2y_2z_2$ at the moment of time t_2, equal t_{2o} (when t_2=0) and corresponding to time moment t_{1o} in system of readout $O_1x_1y_1z_1$, bodies 1 and 2 will be in one point (points S_1, S_2, S and the beginning of coordinates O_2 coincide), coordinate x_{21o} of body 1 and coordinate x_{22o} of body 2 will be equal to zero:

$$x_{21o} = x_{22o} = 0 \qquad (22)$$

speed V_{21o} of body 1 and speed V_{22o} of body 2 will be equal 0:

$$V_{21o} = V_{22o} = 0 \qquad (23)$$

As already it was marked above, in inertial system of readout $O_1x_1y_1z_1$ at the moment of time t_{1o} (t_1=0) and in inertial system of readout $O_2x_2y_2z_2$ at the moment of time t_{2o} (t_2 =0) bodies 1 and 2 will be in one point (points S_1, S_2, S and the beginnings of coordinates O_1 and O_2 coincide), thus the spring 3 will be completely compressed.

Using formulas (19) and (21) it is possible to receive that in inertial system of readout $O_1x_1y_1z_1$ at the moment of time t_{1o} (t_1=0) the system of bodies 1 and 2 (and springs 3) has impulse P_{1o} equal:

$$P_{1o} = P_{11o} + P_{12o} = \frac{2 \cdot M_0 \cdot V}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (24)$$

As the second moment of time represented on fig. 8, in inertial system of readout $O_1x_1y_1z_1$ we will choose the moment of time t_1, equal t_{1t} when the spring 3 from a body 1 is completely unclenched, thus speed V_{11t} of body 1 is equal V (speed of movement of system of readout $O_2x_2y_2z_2$ concerning system of readout $O_1x_1y_1z_1$):

$$V_{11t} = V \qquad (25)$$

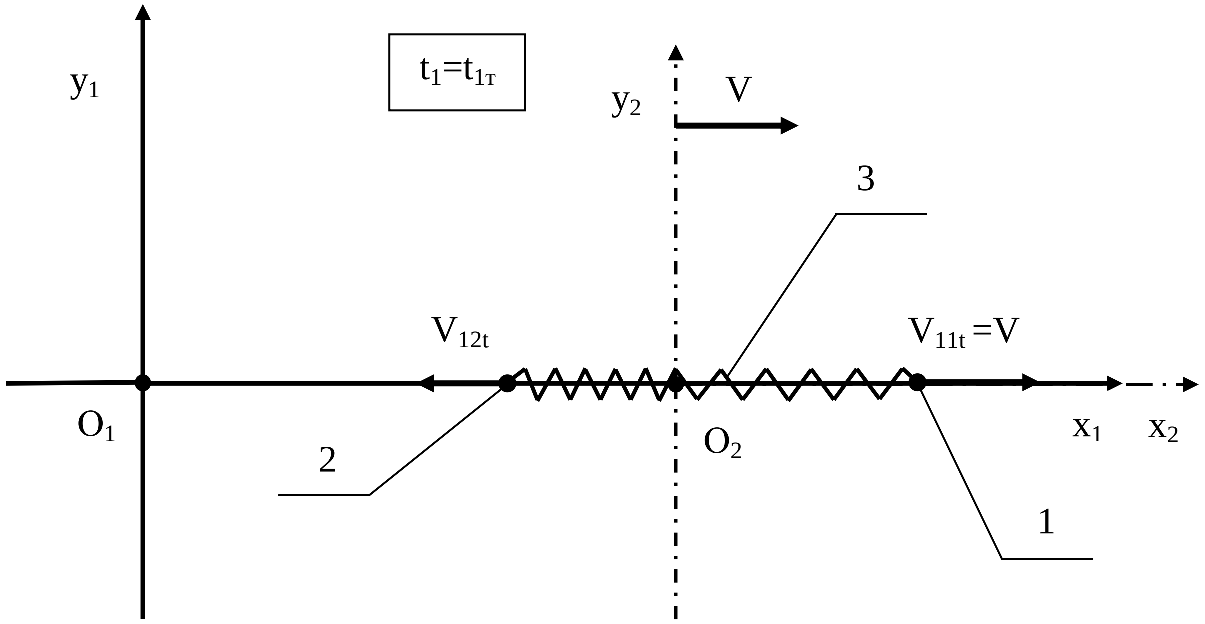

Fig. 8

In inertial system of readout $O_2x_2y_2z_2$ at the moment of time t_2, equal t_{21t} and corresponding to time moment t_{1t} in system of readout $O_1x_1y_1z_1$, coordinate x_{21t} of body 1 will have the maximum value, since at the moment of time t_{21t} the spring 3 is completely unclenched (and has the maximum potential energy of a stretching) and the body 1 - is motionless, thereof speed V_{21t} of body 1 at the moment of time t_{21t} will be equal to zero:

$$V_{21t} = 0 \qquad (26)$$

In inertial system of readout $O_1x_1y_1z_1$ at the moment of time t_{1t} the body 2 has some speed V_{12t}.

In inertial system of readout $O_2x_2y_2z_2$ at the moment of time t_2, equal t_{22t} and corresponding to time moment t_{1t} in system of readout $O_1x_1y_1z_1$, the coordinate x_{22t} of body 2 cannot have the maximum value, since at the moment of time t_{22t} the spring 3 is not completely unclenched (and has no maximum potential energy), hence speed V_{22t} of body 2 at the moment of time t_{22t} cannot be equal speed to zero 0:

$$V_{22t} \# \ 0 \qquad (27)$$

because according to a condition (16) moment of time t_{22t} always should be more moment of time t_{21t}.

From formulas (10) and (27) follows that in inertial system of readout $O_1x_1y_1z_1$ at the moment of time t_{1t} speed V_{12t} of body 2 cannot be equal V (speed of movement of system of readout $O_2x_2y_2z_2$ concerning system of readout $O_1x_1y_1z_1$):

$$V_{12t} \# \ V \qquad (28)$$

Using formulas (17) - (19) and (25), it is possible to receive that in inertial system of readout $O_1x_1y_1z_1$ at the moment of time t_{1t} the system of bodies 1 and 2 (and springs 3) has impulse P_{1t} equal:

$$P_{1t} = P_{11t} + P_{12t} = \frac{M_0 \cdot V}{\sqrt{1 - \frac{V^2}{c^2}}} + \frac{M_0 \cdot V_{12t}}{\sqrt{1 - \frac{{V_{12t}}^2}{c^2}}} \qquad (29)$$

Because the mechanical system of bodies 1 and 2 (and springs 3) is closed, the law of preservation of an impulse allows write down for time moments t_{1o} and t_{1t} in inertial system of readout $O_1x_1y_1z_1$ a following equation:

$$P_{1o} = P_{1t} \qquad (30)$$

or, proceeding from formulas (24) and (29):

$$\frac{2 \cdot M_0 \cdot V}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{M_0 \cdot V}{\sqrt{1 - \frac{V^2}{c^2}}} + \frac{M_0 \cdot V_{12t}}{\sqrt{1 - \frac{{V_{12t}}^2}{c^2}}} \qquad (31)$$

From the equation (31) follows that a necessary condition (value of speed V_{12t}) at which in a considered example the law of preservation of an impulse in inertial system of readout $O_1x_1y_1z_1$ will be carried out, is:

$$V_{12t} = V \qquad (32)$$

or taking into account the formula (9):

$$V_{22t} = 0 \qquad (33)$$

The equations (32) and (33) show that for performance of the law of preservation of an impulse it is necessary, that:

$$V_{11t} = V_{12t} = V \qquad (34)$$

$$V_{21t} = V_{22t} = 0 \qquad (35)$$

And for performance of the equation (35) it is required, that:

$$t_{21t} = t_{22t} \qquad (36)$$

As a result in a considered example in inertial system of readout $O_1x_1y_1z_1$ it is had two requirements contradicting each other:

- The law of preservation of an impulse demands performance of a condition (36),

- The special theory of relativity demands, that the condition (16) arising owing to not of a simultaneity occurring in inertial system of readout $O_2x_2y_2z_2$ of events which in inertial system of readout $O_1x_1y_1z_1$ occur simultaneously was satisfied.

4. The conclusion

In summary it is possible to notice that use of the special theory of relativity by consideration of separate examples can lead to default of the law of preservation of an impulse of the closed mechanical system in inertial systems of readout.

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